THE UNIVERSITY OF ALBERTA

THE EFFECT OF LABORATORY TEACHING METHOD ON ATTITUDES AND ACHIEVEMENTS OF LOW ACHIEVERS IN HIGH SCHOOL MATHEMATICS

by

© JOHN ODYNSKI

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled "The Effect of Laboratory Teaching Method on Attitudes and Achievements of Low Achievers in High School Mathematics," submitted by John Odynski, in partial fulfilment of the requirements for the degree of Master of Education.

ABSTRACT

The purpose of this study was to determine the effect of the laboratory method in teaching Mathematics 15. Tests on student attitudes and achievement were given to assess this effect.

A laboratory approach is defined by student use of concrete materials in acquiring concepts. In this study each directed laboratory lesson began with students doing preliminary exercises which led to a game or an interesting problem. Each non-directed laboratory lesson began with a game or an interesting problem followed by exercises.

Two parallel sets of laboratory booklets (directed and non-directed) were developed on a chapter in Mathematics 15. Each laboratory booklet contained concepts on counting, permutations, combinations and probability.

Six Mathematics 15 classes, two forming the control group and four forming the experimental group, were chosen. Each experimental class was divided into two groups. The groups used directed and non-directed laboratory materials.

At the beginning of the experiment half of the students in each group wrote the attitude test. At the conclusion of the three week experiment all the students wrote the attitude test and the achievement test. The teachers were given the questionnaire after the completion of the experiment.

The Solomon Four-Group Design was used to test for possible interaction between pretest and treatment. The results indicated

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the appearance of interaction between pretest and treatment.

The Wilcoxon Matched-Pairs Signed-Ranks Test and the Mann-Whitney U Test were used to analyze the differences in attitude scores among the various groups of students. The results provided only one significant difference. There was a significant decrease in the post attitude scores of the students in the control group who had written the attitude pretest.

The one-way analysis of covariance with the verbal and non-verbal I.Q. scores as covariates was used to analyze the mean achievement scores among the various groups of students. The results produced no significant differences in mean achievement scores.

In general the teachers favored the laboratory approach over the traditional method in teaching Mathematics 15. However, they did not wish to prepare such laboratory lessons because of the time involved.

This study indicated that the laboratory approach is neither superior nor inferior to the traditional method in teaching Mathematics 15. However, the lessons and materials may be used to change the regular routine of the classroom.

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CHAPTER I

THE EFFECT OF LABORATORY TEACHING METHOD ON ATTITUDES AND ACHIEVEMENTS OF LOW ACHIEVERS IN HIGH SCHOOL MATHEMATICS

Introduction

Mathematics plays an important part in today's technological world. Tomorrow's society will increasingly stress a need for mathematically literate manpower. The understanding of basic concepts in arithmetic, algebra, and geometry is more essential than ever before. Also, since a principal aim of education is to develop a student to his full potential as much as is possible, the low achiever in mathematics is most worthy of careful and considerate professional attention. These are the students who are the potential dropouts. If they remain in school they generally exhibit a pattern of low achievement in mathematics. They will emerge possessing no skills because they are unschooled. That, in turn, leads to unemployment.

According to Johnson and Rising (1967) the low achiever is a student who generally ranks below the thirtieth percentile in achievement for any of the following reasons:

1. Low mental maturity -- This student's I.Q. is usually below 90. He has little ability to perceive relations, is not able to generalize and has difficulty in transferring knowledge.

2. Emotionally immature -- This student comes to school without hope, is hostile to the teacher and the school, is depressed and is frustrated. He usually lacks acceptance, affection, security, and success.

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3. Socially immature - The social and cultural experiences of this student at home, in the community, and in the school are often meager. He often shows aggressiveness and physical restlessness.

4. Psychological deficiencies - This student often does not possess insight, creativity, and problem solving skills. His attention span and memory are short.

5. Meager educational experiences - Previous mathematical experience, study habits and achievement of this student are likely inadequate. Learning has no meaning for him.

In 1965 the United States Office of Education, along with the National Council of Teachers of Mathematics, was concerned about implementing better instructional programs in mathematics for the low achiever. They gave the following reasons:

1. In general, the main reasons for students' dislike of school is due to the lack of both achievement and interest in the instructional program. If these shortcomings are not corrected, they will result in school dropouts.

2. Training in mathematics gives the student a much broader choice in types of occupations; especially training in the late secondary schools or post-secondary technical training institutions. Mathematics will be one of the training needs for skilled and semi-skilled citizens of the future. Two-thirds of the skilled and semi-skilled jobs on the labor market need some mathematics background.

3. The low achiever in mathematics, with the proper program and improved methods of teaching, may be able to enter the labor market less vulnerable to unemployment possibilities.

4. In the past fifteen years, emphasis and attention have been directed toward the above average mathematics achiever and even the average achiever has, to some extent, been neglected. It is felt that if a program designed to raise the mathematical achievement level of the lowest thirty percent was successful then this in turn would affect the average students' achievements as well.

5. There is a positive correlation between success or measurable achievement in mathematics and increased achievement in other disciplines.

In conclusion there seems to be a growing concern among educators for the low achievers in mathematics because they will not possess the necessary skills for employment in our technological world. Also, for personal development, a student's learning experiences should be meaningful and be successful. This should build up a student's self-concept. The traditional method of teaching such pupils does not seem to produce the desired results. Therefore, other teaching methods must be implemented. Otherwise, these pupils will become school dropouts and, subsequently, unemployable. Some people believe the low achieving pupils constitute the most dangerous social condition in society.

The Problem

The low achieving student creates an extremely complex problem whose solution depends on the creative talents not only of the teachers but of other specialists concerned with mathematics education. According to the National Council of Teachers of Mathematics (1968), there is a lack of materials that are particularly appropriate for use with these students. The materials mentioned included textbooks,

visual aids, work sheets and manipulative devices.

Today, there are far too many pupils who dislike mathematics. This dislike seems to increase as they grow older. As a result pupils often encounter great difficulties with content which is really very simple. These attitudes, which were long in building, are carried into the high school mathematics classes. These attitudes have been built over an extended period of time and they are difficult to change. This makes teaching low achievers in high school particularly discouraging and as a result is often rationalized as being unimportant.

It seems that today teachers often are reluctant to teach low achievers. They consider teaching low achievers to be a difficult and often a futile task. For the teacher to encourage learning in the low achiever, the teacher will need appropriate content, materials and methods of instruction.

According to L. K. Johnson (1962), teachers do not command the attention, interest, and reasoning ability of low achievers when they are taught in an abstract, symbolic manner.

Because of the reasons cited above, other methods of teaching mathematics to low achievers must be introduced into our schools. The laboratory approach to teaching mathematics, although still in the experimental stage, has had some success.

From readings of related literature and in the writer's own opinion, it appears that the general objectives of a laboratory approach in teaching mathematics are the following:

1. to arouse interest and increase motivation
2. to allow students to think for themselves

3. to develop an appreciation for mathematics
4. to give the students an understanding of the scientific approach
5. to enable students to build and test theories by themselves
6. to provide for individual differences
7. to develop positive attitudes in students toward mathematics classes
8. to provide continuous reinforcement - hopefully in the form of success, but students also learn by their failures
9. to provide students with meaningful experiences
10. to make teaching low achievers a rewarding experience.

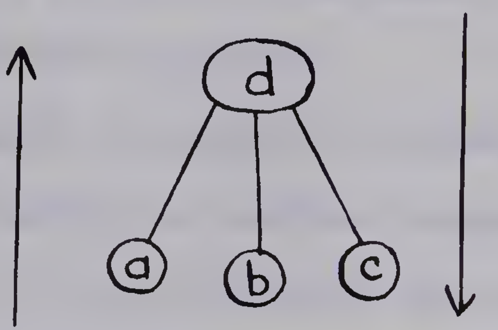
In 1965 the National Council of Teachers of Mathematics suggested that rewards for low achievers must be more obvious, immediate, probable, punishment less threatening, values easier to conceive and perceive. What the low achiever wants now is values for the present and not to look at the future. Immediate reward is the key to the low achiever's success. From readings of related literature and in the experimenter's own opinion, the laboratory method could be a significant factor in providing the appropriate setting for the low achiever as cited above.

There are various ways of structuring laboratory lessons. In this study the laboratory approaches are referred to as being directed and non-directed. In the directed laboratory approach each lesson begins with a tree diagram or some preliminary questions which lead up to a game or a difficult but interesting problem. More questions follow to reinforce the concept. In the non-directed approach each lesson begins with a game or with a difficult but

interesting problem. Questions follow to aid the students in testing their hypotheses about the game or about the initial problem.

In the form of a diagram, the two approaches could be visualized as follows:

Directed



Non-directed

In the directed approach the pupils are acquainted with concepts "a", "b", and "c" which lead up to the main concept "d". In the non-directed approach the students are confronted with concept "d" from which they are to deduce the sub-concepts "a", "b", and "c". The amount of teacher aid is the same for both approaches.

Purpose of the Study

The main purpose of this study was to determine whether the laboratory approach in teaching low achievers in high school would have a positive effect on the students' attitudes toward mathematics classes and on their achievements in mathematics. The effect that the two different laboratory structures had on the low achievers' attitudes toward mathematics classes and on their achievements in mathematics was also of interest to the experimenter. More specifically, an attempt was made in this study to provide answers to the following questions:

1. To what extent did the laboratory method of doing mathematics affect the students' attitudes toward mathematics classes?

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2. Did the type and amount of verbal and written instruction that the pupils received in doing their assignments affect their attitudes toward mathematics classes?
3. How were the students' achievements in mathematics affected by the laboratory approach?
4. Did achievement in mathematics depend on the type of laboratory method the students engaged in?
5. What effect did the laboratory method have on teachers' attitudes toward teaching mathematics to low achievers?

Need For Study

From readings of related literature, there is a world wide growing concern for the low achiever in mathematics. Reports of the United States Department of Labor on retraining of workers indicate that mathematics and reading are the key subjects for making low achieving students employable. According to the National Council of Teachers of Mathematics (1968), there is a lack of textbooks, visual aids, work sheets and manipulative devices that are particularly appropriate for use with these students.

The Mathematics 15 curriculum in Alberta is designed for students who received a "D" or "F" in grade nine mathematics. The present curriculum offers an extensive review of the basic skills. These skills include addition, subtraction, multiplication and division of whole numbers, fractions, and decimals. There are many sections which involve solving problems related to every day situations. Also some elementary aspects of algebra are covered.

By communicating with mathematics teachers who have taught

Mathematics 15 and from the writer's own experience, the teachers find it difficult to teach Mathematics 15 in the traditional manner. Many of the teachers work hard and try to establish a satisfactory rapport with the students in the mathematics classroom. That is, the teachers feel it is their duty to make the learning of mathematics a profitable and an enjoyable experience for these students, but in most cases the teachers' efforts are to no avail. Because of repeated failures, the teachers have become discouraged and show little interest in these students and in teaching them.

One of the requirements of the Department of Education is that students on a diploma program are to take at least one high school mathematics subject. Credits are given for a minimum final mark of forty percent. Because of their previous mathematical experiences, the low achievers often are not interested in studying more mathematics. Since many of the low achievers are on the diploma program, they are forced to take a mathematics course in high school. Since the students are forced to take a mathematics subject, this further frustrates and depresses them. In some cases they become quite hostile toward school and the teacher. Frequently the low achievers miss classes, are inattentive, non-cooperative and refuse to work to their full capacity. Their main goal is to put in as much effort as is required to obtain a final mark of forty percent.

The Department of Education has experimented with various texts in the past four years, but this has not alleviated the problems. The pupils, no matter what text they were using, were still disinterested, missed classes and refused to be cooperative and work to their full capacity.

Although many studies using the laboratory approach have been done, few if any, have involved the low achiever in the high school. For instance, the studies done using the laboratory approach at the University of Alberta have been in the junior high area and in the field of geometry. Furthermore, the 1970 curriculum guide for Mathematics 15 suggests the use of laboratories to be implemented wherever possible. Therefore it seems that there is a need for research to determine the effects of the laboratory method in teaching non-geometric concepts to the low achievers in the high school. In the words of Donovan Johnson:

> Too often our teaching of mathematics has missed its mark and has left the student with a dislike for mathematics rather than an appreciation of the subject. When your students leave your classroom, are they glad they have spent the period with you or do they say, "Aw, all we do in math is work foolish problems"? Do your students want to study mathematics more after being in your classroom a year than before they started? Do they say, "I think math is my most important subject"? It is likely that the greatest residue of our instruction is the attitudes we have nurtured. It is these attitudes which are likely to influence retention, stimulate further study, and interest others in the study of mathematics. And if the right attitudes aren't growing in our classrooms, we need to do something about it (D. A. Johnson, 1957, p. 120).

Delimitations of the Study

Students who receive a "D" or "F" in the grade nine mathematics subject are required to register in Mathematics 15 if they are on a diploma program in high school. Only six classes of Mathematics 15 students plus their teachers in three different high schools in the Edmonton Public School System were involved in this study. Other low achievers in other mathematics courses offered in the high schools were not included in this study.

Nineteen different lessons plus ten sets of review questions were developed to cover approximately three weeks of instruction under the semester system. Since a semester period is 80 minutes per day, these lessons covered approximately 1,200 minutes of instruction.

These lessons covered only one unit on non-geometric material in Mathematics 15 involving concepts on counting, permutations, combinations and probabilities.

Limitations

In this study it was difficult to obtain large random samples. Therefore the conclusions and interpretations reached in this study should be viewed with caution. These conclusions and interpretations are only applicable to the Mathematics 15 student population and not to any other mathematics taught in the high schools.

Since only four teachers were involved in teaching the laboratory method, it was necessary to report their opinions as a case study. Therefore, these results should be viewed with caution in generalizing them to a larger population.

Definitions

Low Achiever: ". . . the student who normally ranks below the 30th percentile in achievement . . ." (Johnson and Rising, 1967, p. 188).

Achievement: an act of achieving or accomplishment

Attitudes: ". . . predispositions to respond toward certain objects, conditions, or events either positively, negatively, or neutrally. Classroom attitudes are dimensions of motivation which

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in turn are directly related to the learning process" (Jones, 1968, p. 604).

Traditional method: a classroom situation in which the teacher was given the freedom to teach in whatever way was considered appropriate by that teacher. The laboratory materials written for this study were not used by the control class teachers.

Mathematical laboratory: a classroom situation which involves the pupil, individually or in groups of two's or three's, in active manipulation of concrete materials.

Directed laboratory: a classroom situation in which pupils solve problems through concrete experiences. Each lesson is introduced by requiring the student to complete a tree diagram or to answer a few preliminary questions which lead up to the game or major problem which follows. Questions follow to reinforce the concept which was covered in the lesson.

Non-directed laboratory: a classroom situation in which pupils solve problems through concrete experiences. Each lesson begins with the pupils either playing a game or trying to solve a difficult but interesting problem. Questions follow to aid the students in testing their hypotheses about the game or about the initial problem.

As much teacher help as was required by the students in both laboratory approaches was to be provided by the teachers.

Outline of the Report

The present chapter consists of an outline and preview of the study. Chapter II contains a review of literature relating to the study. A detailed description of the design of the study

plus the development of the testing instruments and laboratory lessons is given in Chapter III. The analysis of the data is contained in Chapter IV. Chapter V consists of a summary of the investigation, conclusions, and recommendations for further research.

CHAPTER II

REVIEW OF THE LITERATURE

Introduction

The first part of the review is focused on some learning theories and what implications can be drawn from them in the teaching of mathematics. Also included in this part are views of various people associated with mathematics education on the use of concrete materials and their application to the teaching of mathematics. The second part is devoted to the underlying philosophy and rationale for the establishment of mathematics laboratories. The third part is focused on the role attitudes play in the learning of mathematics. Factors which may affect attitudes are also mentioned here. The fourth part is devoted to experimental studies involving the laboratory method in the teaching of mathematics.

Learning Theories and Implications

Recent findings, related to how children learn, support the methods and procedures used in mathematics laboratories. One of the principal goals of any mathematics program is to provide for the development of understanding and meaning. The proponents of mathematics laboratories suggest that the main function of such a teaching method is to provide the learner with an opportunity in which he can form abstract concepts through the manipulation of physical objects. The theories and work of Piaget, Bruner, Dienes and others concerned with mathematics learning have sparked an increased interest in the use of a laboratory approach in the learning of mathematics.

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Piaget: According to Dienes (1960), Piaget was the first to notice that the process of forming any concept takes longer than previously had been believed. In Piaget's learning theory the child's intellectual development is divided into three main stages. In the pre-operational stage or play stage the child is involved in many experiences which may seem unrelated to the concept. However these experiences must be had before thinking takes some direction. Symbolic play can be used by the child to assimilate reality to his own egocentric interests. The child's intellectual functioning operates midway between the generality of a true concept and the particular instances of which it is composed. The child's thought processes are still mainly tied to perceptual appearances but there is a limited amount of conceptual activity.

In the next stage, the stage of concrete operations which occurs from about seven to eleven years, concepts are formed from experiences with real objects. The child's learning should be as free as possible and the ingredients of the concept available as play material. During this stage concept formation is more directed and purposeful but still there is no clear realization of what is being sought. A certain degree of structured activity is desired so that basic concepts are acquired and organized into qualitatively new stable structures. According to M. Adler (1963) concrete operations still have their limitations. The concepts at this stage are still not generalized to all situations. Adler cites the example of conservation of weight and volume as yet to be combined into an organized, structured whole.

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The third stage or the stage of formal operations provides for the practice of fixing and applying concepts that have been formed. At this stage children start to reason by hypotheses rather than in terms of real objects only. According to E. Duckworth (1964), a former student of Piaget at the Institute of Genetic Epistomology in Geneva, Switzerland, these stages are formed and fully constructed when their time comes and there is little we can do to advance them. She interprets the implications of Piaget's theory in the following manner:

> You cannot further understanding in a child simply by talking to him. Good pedagogy must involve presenting the child with situations in which he himself experiments in the broadest sense of that term -- trying things out to see what happens, manipulating things, manipulating symbols, posing questions and seeking his own answers, reconciling what he finds at one time with what he finds at another, and comparing his findings with those of other children (Duckworth, 1964, p. 173).

In general, Piaget's viewpoint on teaching procedures may best be summarized by his comments at the Cornell and California Conferences in 1964. His comments, as reported by Duckworth, were as follows:

> The question comes up whether to teach the structure, or to present the child with situations where he is active and creates the structures himself . . . . The goal in education is not to increase the amount of knowledge, but to create the possibilities for a child to invent and discover. When we teach too fast, we keep the child from inventing and discovering himself . . . . Teaching means creating situations where structures can be discovered; it does not mean transmitting structures which may be assimilated at nothing other than a verbal level . . . . The teacher must provide the instruments which the children can use to decide things experimentally in physics, deductively in mathematics. A ready made truth is only a half-truth (Duckworth, 1964, p. 174).

In general, Piaget's learning theory suggests the manipulation of physical materials before the age of twelve. However his learning

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theory does not suggest the termination of the use of these materials at this age and also it does not distinguish the intellectual development among different types of students. In the words of T. E. Kieren

> Although these age levels have been subject to question, the order of the stages seems fixed and has led to the speculation that learning should proceed from concrete representations to more formal and abstract representations (Kieren, 1971, p. 229).

The present study involved high school low achievers who had not acquired the appropriate attitudes nor the necessary mathematical concepts through the traditional method. It did seem that the use of concrete materials could alleviate some of these problems according to Piaget's theory.

Bruner: Cognitive growth in humans is dependent on the emergence of two forms of competence, according to Jerome Bruner. The two forms of competence are representation and integration. Children, in their developmental process, must acquire ways of representing regularities in their environment as a means for dealing with information. The information is to be organized to enable processing into a higher order of cognitive structure which permits long-range problem solving efforts. These efforts may link the student's past knowledge to the problem he is presently solving and at the same time may give him an insight into solving a more complex problem.

Bruner's learning theory identifies three modes of representation by which a person translates an experience into a model of the world. The first stage is enactive. Here a person represents events through appropriate motor responses or actions. The second stage

he calls iconic which consists of forming images that stand for a concept. In the last stage, symbolic, a student translates his experiences or concepts into words or symbols.

According to Bruner (1960) to have effective teaching we should provide an opportunity for the child to experience concrete manipulations which in turn will lead to the utilization of more adequate modes of thought. If a child in his learning goes through the stages as proposed by Bruner and if he fails in the symbolic transformation, he can fall back on the iconic or enactive stages. If a teacher only presents concepts at the symbolic stage and if the child does not understand the concept, then he will have no other stage to rely on in gaining an understanding of the concept.

Bruner proposes that what is important in teaching concepts is to guide the child progressively from concrete thinking to the use of more conceptually adequate modes of thought. The fundamental structure of a discipline can be presented in such a way as to provide exciting sequences leading the student to discover for himself. In summary, Bruner has the following viewpoint on the teaching of mathematics:

> We reached the tentative conclusion that it was probably necessary for a child learning mathematics, to have not only a firm sense of the abstraction underlying what he was working on, but also a good stock of visual images for embodying them. (Bruner, 1966, p. 66).

According to Ripple (1964), Bruner's and Piaget's learning theories are analogous. Ripple envisions Bruner's developing child incorporating successive emergence of action, image, and word as vehicles of representation. These in turn remind Ripple of the pre-operational, concrete operations, and formal operations as

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suggested by Piaget.

Because Bruner's learning theory does not suggest the age levels at which there is a transfer from the enactive to the iconic to the symbolic stages, it did seem reasonable to teach concepts to Mathematics 15 students at the enactive or iconic levels. Then at least these students would have the iconic and enactive stages to fall back on if they did not understand the concepts at the symbolic level.

According to T. E. Kieren

> Although secondary school children can operate at the symbolic level alone, Bruner sees a danger in simply instructing and learning at this level: if in learning or problem solving symbolic transformation fails a person, he needs to be able to function with appropriate manipulative or iconic transformations (Kieren, 1969, p. 513).

Dienes: Dienes (1960) formulated his theory of mathematics learning in terms of the following four principles:

1. Dynamic Principle -- Necessary experiences must be provided from which mathematical concepts can eventually be built. Play with concrete materials leads to mental games. This, in turn, may give the students an insight into mathematical research.

2. Constructivity Principle -- Children develop constructive thinking long before analytic thinking but analytic thinking is based on construction. Analytic thinking does not normally occur before the age of twelve and until this time the child thinks in terms of actions with real objects.

3. Mathematical Variability Principle -- This principle involves the use of a variety of different variables, some of which may not be essential to the structure, so as to spotlight what is really

constant. The constant feature is the general mathematical concept.

4. Perceptual Variability Principle -- This principle expects the student to draw out the common concept from the different situations he has encountered and to form the concept in abstract terms. The situations are different but contain common properties which will lead to the abstraction to be learnt.

Dienes (1960) was of the opinion that present day methods of teaching mathematics do not incorporate Piaget's learning theory. The mathematics learned is of an associative type. That is, "children associate certain situations with processes, and carry out the processes every time they find themselves in the situations with which the processes have been associated" (Dienes, 1960, p. 45). A strange mathematical situation is created if different letters are used or if the problem is stated in a slightly different manner. In other words, a transfer does not take place since there is no general understanding of the basic mathematical principle.

Because of his learning theory and his experiences in mathematics instruction, Dienes suggests that the use of a considerable amount of concrete material is necessary for learning of mathematics to be as meaningful as possible. With the aid of instructions, the manipulations will provide the children with the appropriate experiences. They will progress in a sequential manner forming a conceptual structure of mathematics in their minds. The structural activities will lead to the formation of abstract concepts. These should be followed by problem exercises which are as practical and meaningful as possible to give assurance that the concepts are truly operational.

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Dienes regards mathematics as a structure of relationships. Symbolism is merely a way of communicating parts of the structure from one person to another. Therefore, according to Dienes, the learning of mathematics is the apprehension of the relationships together with their symbolization. Furthermore, learning of mathematics is the ability to apply the concepts to real situations occurring in the world.

To allow for individual differences learning should take place individually or in groups of two's or three's. Dienes (1960) argues that it is not likely that more than three pupils will work at the same pace and in the same manner. Because of this, all the information cannot come from the teacher. The teacher will not have the time to go around and give individualized instruction to perhaps twenty groups or individual children during a class period, possibly at different stages.

Such a situation will necessitate a change in the role of the teacher. According to Dienes (1960) an authoritarian attitude would hinder a learning situation of this kind, because authoritarianism does not foster a spirit of inquiry. In a creative learning situation there is an atmosphere of keenness to inquire. The teacher must see that the lines of communication from the sources of information to the children are kept open. The teacher is to act as a counsellor and to help the children in their efforts to grapple with the problems in front of them.

At the same time the children must feel the teacher is in charge and that he will help them if necessary.

The teacher's task is easier because of the pupils' interests in

their activities. The teacher must be enthusiastic about such an approach. This should have an effect on the children so that discipline problems ought to diminish. Dienes, who has had experiences with teaching children using the laboratory technique, made the following comment:

> A conventionally trained teacher might wonder how all work is kept going without the force of the authoritative figure at the head of the classroom. Will the children want to do all this, if they are not somehow made to? This is the vexed problem of motivation . . . . If a teacher administering a creative learning-scheme is himself convinced of its rightness, his enthusiasm will be caught by the children and the problem of class discipline will dwindle . . . . We have sufficient evidence now to say that any good teacher who has an easy relationship with his pupils is perfectly able to handle the disciplinary problems of this kind of situation if he is able to handle the other, more usual type of situation (Dienes, 1960, p. 46-47).

In summary, Dienes' position on how to teach mathematics may be summarized by the following comment:

> If the best way of learning something is by doing it, this surely applies to mathematics; we learn by actually doing, constructing mathematics, rather than by going through predetermined sequences where each act prompts the next in a mechanically determined way (Dienes, 1963, p. 165).

In the experimenter's viewpoint most of the low achievers' learnings are of the associative type. They attempt to imitate the teacher or the textbook in their solutions to problems. They do not actually understand the basic mathematical processes involved. According to Dienes, students are to manipulate various physical materials to develop thinking procedures and therefore gain insights into solutions of problems.

Besides the above well-known psychologists and mathematicians, there are other people associated with mathematics education who advocate the use of concrete materials in teaching. Cronbach (1966)

mentioned that the real and valuable knowledge is that formulated by the pupil out of his own experiences.

Picard (1969) stated that talking to the child doesn't result in learning. The child is to be actively involved in creating the mathematics he is to learn. Physical objects are to provide experiences with new content. Pupils will formulate and may at the same time change generalizations which were based on observations. Also, it is important for a child to compare his answers with those of other children. In this way the child will realize the possibility of more than one solution to a problem.

June Dent (1969) stated that we, ourselves, know that we learn best by first-hand experiences. That is, we learn best by doing and seeing for ourselves and by learning in a manner which involves more than our eyes and ears. Once the children have become familiar working with concrete materials we will find a shift in emphasis from teaching mathematics to learning mathematics, from our experiences to the children's and from our world to their world. Such a situation would be gratifying for all concerned. Her reply to questions of when and how long we should use concrete materials in teaching was the following:

> The use of concrete materials helps to continue in a natural manner the learning processes begun in babyhood and gives each child the first-hand experiences which help him to develop a sound knowledge of mathematical concepts. Some teachers have asked when children should stop using concrete materials. The answer is never! Each grade will need concrete materials to illustrate and demonstrate mathematical concepts which occur at their particular grade level. Certainly concrete materials will always be needed in mathematics by slow learners regardless of their age or grade and should never be denied them (Dent, 1969, p. 32).

From the learning theories and opinions cited above it seems that students, who form concepts by the manipulation of physical materials, develop their cognitive thinking in a sequential manner. From the manipulation of the materials the students may notice common properties occurring. Then they may hypothesize and test these hypotheses with the aid of the materials. Finally the students should be able to form concepts at the symbolic level. In general the pupils progress from an elementary play stage to the formation of concepts at the symbolic level.

Mathematics Laboratory -- Philosophy and Rationale

Today many people associated with education believe that books are "too mechanical" and therefore prefer an interchange between human beings. They maintain that books do not open up the various avenues for the students to form concepts. Many books give the rules for doing mathematics instead of displaying the structure of mathematics. Students also should communicate among themselves in forming hypotheses. Therefore it is believed that there is a need for child-centered schools. A more relevant program of social activities should be developed to make more effective use of the child's natural mode of learning.

R. B. Davis (1967) felt that most attempts to improve secondary school mathematics for the low achiever have been unimaginative. He mentioned that "The same content is taught in a more-or-less same old way, but with added fervor and determination" (Davis, 1967, p. 51).

The reason behind the establishment of the Madison Project

was the belief that many children, who learned mathematics by the traditional method, failed mathematics because they were forced into a passive role. Therefore to avoid this danger, a decision was made to almost never lecture and make very little use of required reading of routine material. The use of the laboratory approach to learning mathematics provides a method for telling whether an answer is right or wrong and one that is independent of the teacher and independent of the textbook.

R. B. Davis (1967) gave the following rationale for the establishment of mathematics laboratories.

1. For some students if not for all students, it is assumed that actual perception and actual active manipulations of physical materials contribute to concept formation.

2. Evidence seems to indicate rather strongly that students prefer this method of learning.

3. By observing the child manipulating physical materials, the teacher often gets more insight as to how the child is thinking about a problem than can be obtained by verbal methods.

4. Superficial rote learning is created by traditional verbal methods. This does not seem to assist the child when he is confronted with tasks involving real objects (i.e. in a shop or laboratory situation).

5. Through the use of actual physical objects the child may realize that there may be more than one solution or one right answer to a problem.

6. By using mathematics laboratories a desirable classroom social setting for individualizing instruction is created.

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R. B. Davis gave the following comments about low-I.Q., culturally deprived pupils who were involved in the project:

> Incidentally, in the long run -- i.e., over the year that we worked with this class -- truancy decreased markedly, and parents reported in their children taking an unprecedented interest in school -- for example, by discussing it at meal-time (Davis, 1966, p. 116).

Dienes made the following comments on the Madison Project:

> The motivation comes from the excitement of manipulating a complex structure and finding ways about it and through it. Children certainly appear to enjoy the lessons and there is much to be said for investigating the practicability of such methods in other parts of mathematics (Dienes, 1963, p. 169-170).

L. K. Johnson (1962) felt that the basic philosophy underlying the laboratory method is that students learn better by doing and applying. Many of today's teaching methods of mathematics involve the doing of various unrelated and unreal problems. This results in a student's learning a sequence of steps to obtain the answer. They never really understand the underlying principles or the possible applications of the newly acquired skills. As a result little real learning is accomplished by the student. The laboratory situation enables students to learn by experimentation why certain methods work and how to apply these methods to real and practical problems. Such a situation will provide the student, through his own experimentation and through association with other students on mutual projects, an opportunity to develop new mathematical concepts. In the words of L. K. Johnson:

> The laboratory projects should be an integral part of a planned unit with both teacher and students having full realization of the desired objectives. The work should be largely informal and individual in nature with the teacher

> acting only as a consultant, offering assistance or advice when desired and encouragement when necessary (Johnson, 1962, p. 587).

R. Harris (1963) believes that motivation through the manipulation of physical materials plays an important role in learning. In mathematics teaching, as in any other subject, careful consideration should be given to the motives of the pupils and that the teaching-learning situation should be structured in such a way that these motives are satisfied. His reasons were stated as follows:

> Activities involving the manipulation and concomitant visual preception of different objects and patterns of objects should be encouraged. The cognitive functions are closely related to the kind of sensory stimuli that are presented to children. The mathematics lesson should be planned to allow the fullest expression of this inherent motive of the pupil for exploration of his environment. Omission to do so will lead to frustration and dislike for the subject (Harris, 1963, p. 21).

This is in complete agreement with J. Biggs who made the following comments:

> Multi-model materials, on the other hand, would be expected more reliably to provide the degree of schematic organisation necessary for the accommodation and assimilation of mathematical structures that the child is likely to meet. Motivation would thus be high, and while most children would be expected to have a sound understanding of mathematics, this would be especially true of the less bright . . . (Biggs, 1965, p. 91).

He further adds:

> . . . children taught by multi-model methods appear to have markedly better understanding and attitudes and motivations in arithmetic, this being most marked in boys, and especially in girls, of average and low ability (Biggs, 1965, p. 91).

The Nuffield Project, which was set up in England in 1964 for children ranging in age from five to thirteen years, stressed the procedures in which children learn and not on what or how to teach. In other words, children are to be set free and to make their own

discoveries. Through such a procedure the children will think for themselves and achieve an understanding instead of memorizing facts for doing drills.

In this project child motivation plays a large part in his learning process and at the same time creates a liking for the subject. Motivation is acquired through the direct experiences of the child and no artificial means of rewarding the child are necessary. In other words:

> There remains the whole question of motivation. If a problem is really meaningful to the child then he will be strongly motivated to solve it. It is only when the problem is totally divorced from the "here and now" of the child's personal interest and experience that the unreality obviates genuine motivation . . . . If the problem emerges within the framework of the child's experience, then no artifical device is needed to encourage to work towards a solution (The Nuffield Foundation, 1967, p. 14).

Davis (1964) remarked that students derive intrinsic rewards from solving a problem or discovering an important relationship. Through the discovery method the child has the satisfaction of having his ideas or hypotheses verified. In this way the child is rewarded by being able to tell his classmates and the teacher what he has found out.

A concrete approach permits students to use their senses to see the problem and the results. When a student has experienced reality, then symbolic representation can be introduced. In this way the symbolic representation is made more meaningful.

It would seem that laboratory teaching methods provide for intrinsic motivation. That is, concrete materials stimulate interest and curiosity and clarify the goal of the learning task. Through the manipulation of the materials students obtain data and

form hypotheses which they, themselves, can evaluate by using the physical aids.

In summary, one of the characteristic features of the laboratory method is to give the pupil a purpose for everything he does. The laboratory technique is intended to hold the pupil's attention and at the same time this technique may inspire him. Also through this technique, the pupil may gain some insight into the part that mathematics plays in our daily lives.

Attitudes and Attitude Factors

Attitudes toward mathematics have long been considered of great importance to educators. An attitude can be considered as being partly cognitive and partly affective or emotional. Attitudes usually involve two dimensions. One of the dimensions, direction, is the like or dislike of the subject. The second one is intensity, which refers to how strongly a person feels about his particular formed attitude. Attitudes towards mathematics are composed of intellectual appreciation of the subject and emotional reactions to it.

Intense motives to avoid mathematics are developed by many people. Rhine said, "Now, there is nothing about mathematics itself that could cause people to dislike it; this could occur only as a result of unfortunate experiences with it" (Rhine, 1958, p. 471). A student's failure to achieve at the level of his ability often leads to a depreciation of self-worth. This is accompanied by unhappiness or frustration. When the individual meets with frustration instead of satisfaction, feelings in the form of dislike

towards mathematics often follows.

Chein (1948) said that a person is not born with his attitudes. The learning process he is involved in plays a major role in the development of attitudes. Attitudes involve problems of perception and motivation and as a result of a particular attitude a person may be more likely to perceive certain objects than others. Chein mentioned that previous learning experiences play a role in determining how a person perceives a given situation or object. What is more important is what the person wants in the situation or with regard to the object. This then determines the attitudes which are generated and influences the already existing attitudes.

D. A. Johnson (1957) believes that learning involves emotional vectors such as attitudes. Attitudes that the students develop will likely stimulate or stop further study of mathematics. Classroom lessons that are dull and uninspiring or homework that is meaningless drudgery will not promote an attitude of appreciation because we learn according to our reactions to experiences. If the students are to like mathematics, they must find pleasure in performing the learning activities involved in mathematics. Students will gain pleasure only if they can do it successfully and that which seems significant in meeting their needs. In the words of Johnson:

> Thus, the student with the proper attitudes will enter wholeheartedly into the learning activities because he is sensitive to mathematics wherever he finds it and derives pleasure from his contacts with it. If the attitudes are only partially developed he will participate in some situations and not in others (Johnson, 1957, p. 118).

The concern for a child's attitudes was one of the factors for the establishment of the Nuffield Project. The project claims

that a child's attitudes towards mathematics are formed in the primary school. The emphasis is on children's enjoyment and success and the prevention of remarks such as the following:

> I was never any good at maths.
> I hated arithmetic.
> Maths always terrified me. (The Nuffield Foundation, 1967, p. 5)

In order to ensure a good attitude towards mathematics a child must have a real understanding and insight into the problem involved and the possible ways it might be approached.

Gough (1954) adds that "mathemaphobia" is caused by fear and dislike for the subject which is caused by previous experiences in mathematics classes.

Experimental Research

The results of a number of studies indicate the persistence of negative attitudes toward mathematics as students progress through school. Aiken (1970) reported that the junior high years are a period when student attitudes toward arithmetic reached a peak of development. Various studies indicate different factors affecting attitudes toward mathematics.

The studies done by Young (1932), Dutton (1968), Chase (1947) and by Dutton and Blum (1968) indicated that reasons for a student losing interest in a subject were attributed to some of the following factors: (1) failure to see a need for the course; (2) uninteresting material; (3) monotonous methods; (4) too difficult. Dutton further commented that liking arithmetic effects the amount of work attempted, the effort expended and the acquired learning. He concluded that lasting attitudes toward arithmetic are developed

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at each grade level but are more pronounced at the grade five and grade seven levels.

If attitudes were not thought to affect performance in some fashion, then the assessment of attitudes toward mathematics would be of less concern. It is thought that attitudes affect achievement and in turn achievement affects attitudes. The above statements are supported in studies done by Kurtz and Swenson (1951) and by Degnan (1967).

Kurtz and Swenson (1951) divided 200 pupils on the basis of both intelligence and achievement into five groups. The groups were: (1) high achievers of high ability; (2) medium achievers of medium ability; (3) low achievers of low ability; (4) "plus achievers" -- students whose achievement was above expectations on the basis of ability ratings; and (5) "minus achievers" -- students whose achievements were below expectations on the basis of ability ratings. The experimenters then questioned the teachers, the students and the parents on the following: (1) how the student was getting along in school; (2) how the student liked school; (3) how important was school achievement to the student; and (4) how important was obtaining an education to the student. The results indicated a definite interrelationship between attitudes towards school, achievement, and an education with actual achievement. Student attitudes toward educational achievement, the school situation, successful school performance, and the importance of an education were more closely related to the students' achievement scores than to their ability scores. High attitude ratings and high achievement

were related while low attitude ratings were related to low achievement.

Studies done by Aiken and Dredger (1957) and by Garner (1963) indicated that pupils' experiences with former mathematics teachers are related to their present attitudes. Garner found significant relations between (1) the teacher's background in mathematics and the achievement of the students in algebra, (2) the attitude toward algebra of the teacher and the students, and (3) teacher's judgements concerning the practical value of algebra and the students' judgements.

According to Poffenberger and Norton (1959) parents affect the child's attitude and performance in three ways. The three ways are: (1) parental expectation of their child's achievement; (2) parental encouragement regarding the subjects; and (3) parental attitudes toward the curricula. Poffenberger and Norton found that parental encouragement to take mathematics courses in high school was significantly related to student attitudes toward mathematics. Also, pupils' attitudes and performance in arithmetic and mathematics are affected by teachers. That is, the pupils' attitudes and achievements depend on previous arithmetic and mathematics experiences in the classroom. Poffenberger and Norton made the following comment:

> Attitudes are developed in the home in some cases before the child begins school. In the first and second grades he is affected not only by his teacher and his readiness to deal with numbers, but also by the attitude of his parents toward the subject matter. He carries into his high school mathematics classes attitudes that are long in building and difficult to change. The fact that so many high school students have negative attitudes toward mathematics makes the job of the teacher doubly difficult and indicates the need for outstanding

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> teachers in terms of knowledge of subject matter, teaching ability, personality, and understanding of adolescents (Poffenberger and Norton, 1959, p. 175).

Since this study is concerned with the effect that the laboratory approach has in the teaching of mathematics on student attitudes and achievement, studies related to this aspect are summarized below.

In the study by J. Vance (1969) the effects of a mathematics laboratory program were investigated. The fourteen grade seven and eight classes in Edmonton were divided into three groups. Once a week the students in one group were to develop a concept with the aid of some concrete materials manipulation. In the other group the teacher discussed the concept with the aid of the materials. The third group served as a control group.

The results indicated no adverse effects in achievement by the experimental groups. The resulting differences in attitude measures, though not significant, favored the group which manipulated the materials. Tests of immediate learning, cumulative achievement, higher level thinking and problem solving and divergent thinking indicated that the pupils in the two types of experimental classes benefited mathematically.

Swick in 1953-54 conducted a study to find out if the use of multi-sensory concrete teaching aids would have any effect on student achievement, student attitude and teacher attitude. The results indicated an improvement in attitude toward arithmetic of second and third grade pupils. Also revealed were better teacher attitudes toward arithmetic and the continued use of multi-sensory aids.

Plank (1950) taught twenty children ranging in age from five to

twelve years and ranging in ability from high to retarded by the use of Montessori materials. She observed that in working with these materials the pupils exhibited a long span of interest and great perseverance. The pupils concentrated for longer periods of time than would usually be planned for and asked of young children. The material was equally interesting to the young as well as to the accelerated and retarded groups. It relaxed and challenged the children's thinking at the same time.

The results of an experiment by Natkin (1966) suggested that to get an individual to associate mathematics with something pleasant may improve his attitude and at the same time make him less anxious with respect to the subject. According to Aiken,

> Many writers (e.g. Lerch, 1961; Tulock, 1957) have observed that pupils who consistently fail in mathematics lose self-confidence and develop feelings of dislike and hostility toward the subject. To cope with such negative attitudes, the teacher must provide success experiences for the learner; the child should be taught to set reasonable goals that culminate in the reward of success (Aiken, 1970, p. 586-87).

He further adds:

> In both the development and modification of attitudes, and in training and remedial work, a question is how to make mathematics more interesting. New methods may be initially motivating, but their effects will not last if the teachers are poorly trained, the parents not sympathetic, and the students are not successful in mastering the subject (Aiken, 1970, p. 591).

In summary, many learning theorists would suggest that positive motivation is a result of rewarded experiences. These experiences, successful or not, may be related to the student's ability. However, it seems that other sources of motivation affect the pupils' attitudes. One of these is achievement. The other factors involve

previous mathematical experiences and parental and peer approval.

Since the traditional method of teaching Mathematics 15 does not alleviate the problems as stated in Chapter I, other methods of teaching these pupils must be implemented. The laboratory approach seems to provide for meaningful learning experiences which in turn should lead to successful experiences. These should affect the students' attitudes toward mathematics classes in a positive manner.

The present chapter included the review of the literature related to this study. Included were: (1) some learning theories and implications; (2) the philosophy and rationale in establishing mathematics laboratories; (3) role attitudes play in learning mathematics; and (4) experimental research related to this study. The experimental design and statistical procedures that were followed in this study are explained in the next chapter.

CHAPTER III

EXPERIMENTAL DESIGN AND STATISTICS USED

Introduction

As stated in Chapter I, the purpose of this study was to compare the attitudes toward mathematics classes and achievements of Mathematics 15 students using the laboratory method and the traditional method of teaching mathematics. Also, the attitudes toward mathematics classes and achievements of the students using the two different laboratory approaches was investigated. Finally the teachers using the laboratory approach were asked to compare teaching Mathematics 15 by the laboratory method and the traditional method. These comparisons were analyzed in the mode of a case study.

In the remainder of this chapter the experimental design that was used is presented. This includes the tests and instruments that were constructed and the procedures used to establish their validity and reliability. The instructions and procedures that were followed in the laboratory classes are described. The sampling procedures are also included. The last part of this chapter is devoted to the null hypotheses being tested and the statistics used to test these hypotheses.

Tests and Instruments

Intelligence Tests: The Lorge-Thorndike Level 5, Form A Verbal and Nonverbal Batteries were used to determine the students' Intelligence Quotients (I.Q.'s). The raw scores on these tests were converted to the respective I.Q. scores by using the tables

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in the Examiner's Manual. The writer either tested a student himself or obtained his I.Q. score from the student's cumulative record which is stored in the school files.

Attitude Test: In order to determine a student's attitude toward his mathematics class, the writer, in cooperation with his colleagues, developed a suitable attitude scale. After considering some of the various possible attitude scales, a semantic differential type of scale was chosen.

Each item on the test involved two words which were antonyms. Five spaces between the two words were provided for the student to mark his choice. Each student writing this test was encouraged to mark in one of the five spaces for each word pair. The weights of the choices ranged from five to one; with the most favorable choice assigned a weight of five and the least favorable choice assigned a weight of one. Which word in each word pair represented the favorable or unfavorable attitude was judged by the designers of the scale. The sum of the weights of a student's choices represented his score on the scale.

In constructing the attitude scale, the writer examined various attitude scales and along with his colleagues determined what were considered positive and negative attitudes toward mathematics classes. Some terms from previous attitude scales were incorporated into this scale. Special attention was paid to the terminology used so as to make the scale as understandable as possible to the Mathematics 15 students.

During the pilot study at M. E. Lazerte High School, the Mathematics 15 students were given an attitude test consisting of forty-four word pairs and after they completed some of the laboratory lessons they were given the same attitude scale again. Using the median as a guideline, the pretest scores were divided into two groups; high and low attitude. The responses of these two groups to each word pair was recorded. The same was done for the posttest responses.

The experimenters then examined the word pairs to determine which word pairs discriminated between the high and low attitude groups. In other words the experimenters chose the word pairs in which the high attitude group scored favorably and the low attitude group scored unfavorably. The original forty-four word pairs were reduced to the best twenty-five word pairs in view of the experimenters' opinions.

A test-retest procedure was used to establish the reliability of the attitude test which now consisted of twenty-five word pairs. At the beginning of the project, two classes of Mathematics 15 students at Eastglen Composite High School in the Edmonton Public School System were given the attitude test. Two weeks later they were given the same attitude test again.

To establish the reliability of each word pair on the attitude test, the writer compared each student's two responses to each word pair. Then the reliability for each word pair was calculated as follows:

> The number of student responses on the posttest that were within plus or minus one of their responses on the pretest divided by the total number of students responding to the word pair.

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The reliabilities of the word pairs ranged from 0.73 to 1.00. Of the 25 word pairs on the attitude scale, three had a reliability ranging from 0.70 to 0.79, six had a reliability ranging from 0.80 to 0.89 and sixteen had a reliability ranging from 0.90 to 1.00 inclusive. The mean of the reliabilities of the twenty-five word pairs was 0.89. A copy of the attitude test used may be found in Appendix A of this thesis.

Achievement Test: Again the experimenters involved in this study constructed the achievement test which consisted of thirty multiple choice questions involving concepts on counting, permutations, combinations and probabilities. Each question consisted of four possible choices from which the student was to choose one and mark his response on a separate answer sheet. The answer sheets were then machine scored.

Each student was required to finish this test within an 80 minute period. All of the students completed the test within this period.

The class mean for the six classes was 15.59 out of a maximum of 30 and the KR-20 reliability was 0.6718. In determining the KR-20 reliability an individual either passes or fails the item. One is assigned for a pass and zero is assigned for a failure. The number of items done correctly determines the score. If p_i is the proportion of individuals passing item "i" and $q_i = 1 - p_i$ is the proportion of individuals failing, then the reliability is determined by

$$r_{xx} = \frac{n}{n-1} \quad \frac{s_x^2 - \sum_{i=1}^{n} p_i q_i}{s_x^2}$$

where n = number of test items

s_x^2 = variance of scores on test defined as

$\sum(x - \bar{x})^2/N$

$p_i q_i$ = product of proportion of passes and fails for item i.

$\sum_{i=1}^{n} p_i q_i$ = sum of these products for n items.

The achievement test may be found in Appendix B.

Teacher Questionnaire: The writer in cooperation with the other experimenters involved in this study constructed this questionnaire. It contained fifteen questions which required the teachers to compare the laboratory teaching method and the traditional teaching method in Mathematics 15. There was also a section provided for any comments the teachers wished to make. The teacher questionnaire and the teachers' responses to this questionnaire may be found in Appendix C of this thesis.

Laboratory Lessons: Two different sets of laboratory lessons involving the concepts of counting, permutations, combinations, probabilities, dependent and independent events were developed by the experimenters. These two different sets of laboratory lessons were referred to as directed and non-directed. For the reasoning underlying these two types of laboratory approaches refer to Katherine McLeod's study.

Each laboratory booklet which was used by a pair of students contained nineteen laboratory exercises, four of which were optional. Each section contained review questions which were to reinforce the concepts covered. A final review, consisting of twenty questions, was included at the end. The review covered all the concepts that were included in the laboratory lessons. Each section was printed on different colored paper for variety and to make it easier for the pupil to find the page he worked on last.

The following is a brief resume of the various sections in the laboratory booklets:

1. Introduction - two lessons on orange paper
2. Counting - four lessons (two of these optional) on green paper
   - two sets of review questions on white paper
3. Permutations and Combinations - six lessons (one of these optional) on pink paper
4. Probability - three lessons on blue paper
   - two sets of review questions on white paper
5. Dependent Events - two lessons on yellow paper
6. Independent Events - two lessons on orange paper
   - two sets of review questions on white paper
7. Final Review - twenty questions on the above concepts

The most important point in the design of each lesson was the requirement that pupils play a game or answer a difficult but interesting problem or do both. There were two or three different activities related to the same concept. The reasoning was that if some students

were having problems doing an activity they might have more success with a different activity which involved the same concept. This is in complete agreement with Dienes' and Biggs' viewpoint.

The optional lessons were developed for the pupils who finished the regular laboratory lessons early. This would keep the pupils in the class at approximately the same place so that the teacher could conduct a class discussion involving all of the students if necessary. At the same time these optional lessons could further reinforce the learning of these concepts.

Large wall charts were developed so that the pupils could check off the lessons they had completed. At a glance, the teacher could determine which groups of pupils were encountering problems. The teacher could also encourage the slower pupils or absentees to complete the lessons that they had not finished.

A sample of the directed laboratory lessons may be found in Appendix D and a sample of the parallel non-directed laboratory lessons may be found in Appendix E of this thesis.

Teacher's Guide: A teacher's guide was written by the experimenters to aid the teachers in assuming their expected role when teaching by the laboratory method. The following is a brief description of the topics included in the guide.

1. The reason for the development of the project by the experimenters.
2. Procedures expected to be followed by the teachers and the students.
3. Suggestions as to how the teachers should introduce the

material.

4. Suggested timetable for the teachers to follow.

A sample of the teacher's guide may be found in Appendix F of this thesis.

Sampling

Six classes of Mathematics 15 students from three different high schools on the semestered system in the Edmonton Public School System were chosen to participate in this study. The three schools were Queen Elizabeth Composite High School, Victoria Composite High School and Strathcona Composite High School. Two of the six participating teachers were chosen at random to teach their respective classes in a traditional manner. The other four teachers taught their classes by using the laboratory booklets that were developed by the experimenters.

The reason for choosing three different schools for this project was to have a better representation of the entire Mathematics 15 population. Finally, since the semestered system is becoming more prevalent in the Edmonton Public School System and since it seemed that teachers were having a more difficult time teaching these pupils for 80 minutes a day, it was decided to choose high schools which were on a semestered system.

Procedure

Pilot Study: A partial pilot study was conducted at M. E. Lazerte High School in the Edmonton Public School System. The experimenters obtained information concerning the clarity of the

attitude test plus the suitability of the activities in the first twelve laboratory lessons.

A more thorough pilot study was conducted by Katherine McLeod at Victoria Composite High School. This pilot study involved twelve Mathematics 15 students chosen randomly from a class of thirty. From this study the experimenters corrected the noticeable errors.

Main Study: As was mentioned before, four of the six chosen Mathematics 15 classes used the laboratory approach. In each of these four classes the pupils were subdivided randomly into two different groups. Half of the pupils in each class used the directed laboratory approach while the other half used the non-directed approach. For further information on these two laboratory approaches refer to Katherine McLeod's study.

Pupils in these classes worked in groups of two or three. They were able to choose their own partners at the beginning of the study and these groups remained intact for the remainder of the study.

To check for possible interaction of pretesting and the treatment a Solomon Four-Group Design was used. From Campbell and Stanley (1966) the design in symbolic form is as follows:

(1)	R	0_1	X	0_2
(2)	R	0_3		0_4
(3)	R		X	0_5
(4)	R			0_6

"R", "0", and "X" respectively represent random sampling, testing, and treatment.

The students in groups (2) and (4) were in the control classes. Half of them were given the attitude pretest but all of them were given the same attitude scale as a posttest. The students in groups (1) and (3) were in the experimental classes. Half of the students in the directed and non-directed classes were given the attitude pretest but all of these pupils were given the attitude posttest. Students in each of the groups were randomly assigned to take the pretest.

By using the above design the main effects of testing and the interaction of testing and the treatment could be determined.

> In this way, not only is generalizability increased, but in addition, the effect of X is replicated in four different fashions: $O_2 > O_1$, $O_2 > O_4$, $O_5 > O_6$, and $O_5 > O_3$. The actual instabilities of experimentation are such that if these comparisons are in agreement, the strength of the interference is greatly increased (Campbell and Stanley, 1966, p. 25).

At the end of the three week period of laboratory lessons, all of the students involved in this study were given the achievement test. Also at this time the four teachers involved in the laboratory method were given the questionnaire.

Before the experiment began the experimenters met with the teachers involved and discussed with them the expected procedures. Consequently, the teachers were acquainted with their role and with the procedures expected of them in teaching these students using the laboratory method.

In this study the same question and review sheets were given to the students involved in the laboratory and the traditional classes. The reason for this was to control for the Hawthorne Effect. In other words the pupils in the control classes were to

feel that they were part of the experiment by doing some of the things that the pupils in the laboratory classes were doing. Also these question and review sheets were used as a control to insure that approximately the same concepts were covered in all of the classes involved in this experiment.

Data Collected

The following data were collected in this study:

1. Lorge-Thorndike verbal and nonverbal I.Q. scores for each pupil.
2. Attitude scores on the pretest and posttest.
3. Achievement scores for each pupil.
4. Teachers' reactions toward teaching Mathematics 15 students by the laboratory method.

Research Questions and Analyses Used

I. Test for Interaction Between Pretest and Treatment

1. The Wilcoxon Matched-Pairs Signed-Ranks Test was used to determine if there were any differences in attitude pretest and posttest scores of the experimental group who wrote both tests.

2. The Mann-Whitney U Test was used to determine if there were any significant differences between the following:

(a) posttest attitude scores of the experimental and control groups who also wrote the attitude pretest.

(b) posttest attitude scores of the experimental and control groups who did not write the attitude pretest.

(c) posttest attitude scores of the experimental group

who did not write the attitude pretest and pretest scores of the control group who wrote the attitude posttest.

II. Research Hypotheses

This study was concerned with five main questions as was stated in Chapter I. These questions are again listed with their corresponding hypothese which were tested. Also, the statistical analyses used to test the hypotheses are given.

Question 1

To what extent did the laboratory method of doing mathematics affect the students' attitudes toward mathematics classes?

Null Hypothesis 1(a):

There is no significant overall difference in the distributions of attitude scores toward mathematics classes between the students using the laboratory approach and the students using the traditional method.

In reference to the Solomon Four-Group Design which is as follows:

(1)	R	0_1	X	0_2
(2)	R	0_3		0_4
(3)	R		X	0_5
(4)	R			0_6

null hypothesis 1(a) can be stated as $0_2 = 0_4$ and $0_5 = 0_6$.

Null Hypothesis 1(b):

There is no significant difference in the distributions of attitude scores toward mathematics classes between students using the directed laboratory approach and the students using the traditional method.

In this case the Solomon Four-Group Design can be stated as follows:

(1)	R	0_1	directed	0_2
(2)	R	0_3		0_4
(3)	R		directed	0_5
(4)	R			0_6

and hypothesis 1(b) is then $0_2 = 0_4$ and $0_5 = 0_6$.

Null Hypothesis 1(c):

There is no significant difference in the distributions of attitude scores toward mathematics classes between students using the non-directed laboratory approach and the students using the traditional method.

Here the Solomon Four-Group Design can be stated as follows:

(1)	R	0_1	non-directed	0_2
(2)	R	0_3		0_4
(3)	R		non-directed	0_5
(4)	R			0_6

and hypothesis 1(c) is then $0_2 = 0_4$ and $0_5 = 0_6$.

All of the above hypotheses were tested by using the Mann-Whitney U Test.

Question 2

Did the type and amount of verbal and written instruction that the pupils received in doing their assignments affect their attitude toward mathematics classes?

Null Hypothesis 2(a):

There is no significant difference in the distributions of attitude scores toward mathematics classes between students using the directed laboratory approach and the students using the non-directed laboratory approach.

Again the Solomon Four-Group Design can be stated as follows:

(1) R 0_1 directed 0_2

(2) R 0_3 non-directed 0_4

(3) R directed 0_5

(4) R non-directed 0_6

and hypothesis 2(a) is then $0_2 = 0_4$ and $0_5 = 0_6$.

The Mann-Whitney U Test was used to test the above hypothesis.

Null Hypothesis 2(b):

There is no significant difference in the gains of the attitudes scores toward mathematics classes within each of the three groups of students.

Using only the students in each group who took the attitude pretest and posttest, we can state this hypothesis as follows:

(1) 0_1 directed 0_2

(2) 0_3 non-directed 0_4

(3) 0_5 0_6

and then hypothesis 2(b) is $0_2 = 0_1$, $0_4 = 0_3$ and $0_6 = 0_5$.

The Wilcoxon Matched-Pairs Signed-Ranks Test was used to test hypothesis 2(b).

Question 3

How were the students' achievements in mathematics affected by the laboratory approach?

Null Hypothesis 3(a):
When the effects of student I.Q. scores are subtracted, there is no significant difference in mean achievement scores between students using the laboratory approach and the students using the traditional method.

Null Hypothesis 3(b):
When the effects of student I.Q. scores are subtracted, there is no significant difference in mean achievement scores

between the students using the directed approach and the students using the traditional method.

Null Hypothesis 3(c):
When the effects of student I.Q. scores are subtracted, there is no significant difference in mean achievement scores between the students using the non-directed approach and the students using the traditional method.

The above three hypotheses were tested by using a one-way analysis of covariance with the verbal and nonverbal I.Q. scores as covariates.

Question 4

Did achievement in mathematics depend on the type of laboratory method the students engaged in?

Null Hypothesis 4(a):
When the effects of student I.Q. scores are subtracted, there is no significant difference in mean achievement scores between the students using the directed approach and the students using the non-directed approach.

This hypothesis was tested by using a one-way analysis of covariance with the verbal and nonverbal I.Q. scores as covariates.

Question 5

What effect did the laboratory method have on teachers' attitudes toward teaching mathematics to low achievers?

Since only four teachers were involved in using the laboratory approach, this hypothesis was treated as a case study. For the teachers' responses to the questionnaire refer to Appendix C in this thesis.

The present chapter included a detailed explanation of the tests developed and used, the sampling procedure, the experimental

design and the hypotheses to be tested. In the next chapter findings of the investigation are presented.

CHAPTER IV

FINDINGS OF THE INVESTIGATION

Introduction

In this chapter an analysis of the interaction between pre-testing and treatment is given via the Solomon Four-Group design. Following this the findings of the study are presented. Data relating to the corresponding hypothesis are presented and analyzed under the five main questions of the study.

The Solomon Four-Group design in symbolic form is as follows:

(1)	R	0_1	X	0_2
(2)	R	0_3		0_4
(3)	R		X	0_5
(4)	R			0_6

where 0_1 and 0_2 are the attitude scores of the students in the experimental group who wrote both the attitude pretest and posttest. 0_3 and 0_4 represent the attitude scores of the students in the control group who wrote both the attitude pretest and posttest. 0_5 and 0_6 represent respectively the attitude scores of the students in the experimental group and control group who wrote only the attitude posttest.

The effect of the pretest can be determined by the following four comparisons: (1) 0_2 and 0_1; (2) 0_2 and 0_4; (3) 0_5 and 0_6; and (4) 0_5 and 0_3. In null form the following hypotheses were tested: (1) $0_2 = 0_1$; (2) $0_2 = 0_4$; (3) $0_5 = 0_6$; and (4) $0_5 = 0_3$.

The Wilcoxon Matched-Pairs Signed-Ranks Test was used to test null hypothesis 1 ($0_2 = 0_1$). This test utilizes both the relative

[illegible]

magnitude as well as the direction of the differences within pairs. In using this test the differences between each pair of scores is determined. These differences are then ranked without regard to the sign. Each rank is then assigned the sign of the difference. The negative ranks and the positive ranks are summed. A "T" score represents the smaller sum of the like-signed ranks. For a sample size greater than 25, a "Z" score is calculated.

The Mann-Whitney U Test was used to test the other three null hypotheses. This was used to test whether two independent groups have been drawn from the same population. In determining U both group scores are combined and ranked. Then

$$U = n_1 n_2 + \frac{n_1(n_1 + 1)}{2} - R_1$$

and also

$$U = n_1 n_2 + \frac{n_2(n_2 + 1)}{2} - R_2$$

where n_1 = sample size of smaller group

n_2 = sample size of larger group

R_1 = sum of ranks whose sample size is n_1

R_2 = sum of ranks whose sample size is n_2.

When the larger of the two samples contains more than 20 scores, a "Z" score is determined.

Following is a summary of the results of the four comparisons.

TABLE I

COMPARISON I

NULL HYPOTHESIS: $0_2 = 0_1$

Group	N	Z	Sig. Level
0_1	35	-1.324	.093
0_2	35		

TABLE II

COMPARISON II

NULL HYPOTHESIS: $0_2 = 0_4$

Group	N	U	Z	Sig. Level
0_2	35	127.5	-3.098	.001
0_4	16			

TABLE III

COMPARISON III

NULL HYPOTHESIS: $0_5 = 0_6$

Group	N	U	Z	Sig. Level
0_5	35	248	-.307	.378
0_6	16			

TABLE IV

COMPARISON IV

NULL HYPOTHESIS: $0_5 = 0_3$

Group	N	U	Z	Sig. Level
0_3	16	219.5	-1.229	.097
0_5	35			

From the results in Table II, the second null hypothesis ($0_2 = 0_4$) can be rejected. However, the results in the other three tables dilute the results in Table II. These three tables suggest that there was some interaction between the attitude pretest and the treatment. Therefore one must be cautious in the interpretation of the results which follow because of the indication of interaction between the attitude pretest and the treatment.

Analysis of Attitude Scores

The purposes of the study were threefold. First, the effect of the laboratory approach on the students' attitudes toward their mathematics classes was assessed. Secondly, the effect of the laboratory approach on the students' achievements was determined. Thirdly, the teachers' opinions on the laboratory approach was examined.

Analysis of the data related to the five questions appear on the following pages. The first two questions are concerned with the first purpose of the study. Tables VII to XI contain these

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[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]

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results. The results in each table will only be significant when the results of all the parts in each table are significant. The next two questions are concerned with the second purpose. The last question pertains to the third purpose.

Tables V and VI contain the range of the attitude scores and the means of the attitude scores respectively within the various groups of students in this experiment. The maximum score on the attitude test is 125. It should be remembered that only one-half of the students in the sample wrote the attitude pretest while all of the students in the sample wrote the attitude posttest.

TABLE V

RANGE OF ATTITUDE SCORES

WITHIN VARIOUS GROUPS

Group	Range on Pretest	Range on Posttest
All groups	56 - 118	26 - 120
Control	71 - 99	26 - 104
Experimental	56 - 118	56 - 120
Directed	62 - 118	56 - 120
Non-Directed	56 - 115	57 - 116

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[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]

TABLE VI

MEANS OF ATTITUDE SCORES

WITHIN VARIOUS GROUPS

Group	Mean on Pretest	Mean on Posttest
All groups	85.2	83.4
Control	82.3	75.2
Experimental	86.6	88.4
Directed	89.3	91.3
Non-Directed	83.3	84.6

Following is the analyses of the five questions with the respective null hypotheses as posed above.

Question 1

To what extent did the laboratory method of doing mathematics affect the students' attitudes toward mathematics classes?

Null Hypothesis 1(a):

There is no significant difference in the distributions of attitude scores toward mathematics classes between students using the laboratory approach and the students using the traditional method.

The Mann-Whitney U Test was used to test this hypothesis. The attitude scores of the pupils who wrote both the attitude pretest and the posttest in the experimental and control groups were compared. Also the attitude scores of the pupils who wrote only the attitude posttest in the experimental and control groups were compared.

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[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]

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The results are summarized in Table VII.

TABLE VII

SUMMARY OF ANALYSIS OF DATA COMPARING

EXPERIMENTAL AND CONTROL GROUPS

Group	N	U	Z	Sig. Level
Exp. (Pre-Post)	35	127.5	-3.098	.001
Con. (Pre-Post)	16			
Exp. (Post)	35	248	-.307	.378
Con. (Post)	15			

Since both parts for testing the null hypothesis 1(a) did not produce significant differences and since there was an appearance of interaction, the null hypothesis was not rejected. In other words, there was no significant difference in the distributions of attitude scores toward mathematics classes between the students using the laboratory approach and the students using the traditional method.

Null Hypothesis 1(b):

> There is no significant difference in the distributions of attitude scores toward mathematics classes between students using the directed laboratory approach and the students using the traditional method.

To test this hypothesis the Mann-Whitney U Test was used. The attitude scores of the pupils who wrote both the attitude pretest and posttest in the directed laboratory and control groups were compared. Also the attitude scores of the pupils who wrote only

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the attitude posttest in both groups were compared. The results are summarized in Table VIII.

TABLE VIII

SUMMARY OF ANALYSIS OF DATA COMPARING DIRECTED AND CONTROL GROUPS

Group	N	U	Z	Sig. Level	Critical U
Dir. (Pre-Post)	19	58			≤ 82
Con. (Pre-Post)	16				
Dir. (Post)	21	142.5	-.482	.3156	
Con. (Post)	15				

Since both parts for testing the null hypothesis 1(b) did not produce significant differences and since there was an appearance of interaction, the null hypothesis was not rejected. In other words, there was no significant difference in the distributions of attitude scores toward mathematics classes between students using the directed laboratory approach and the students using the traditional method.

Null Hypothesis 1(c):

> There is no significant difference in the distributions of attitude scores toward mathematics classes between students using the non-directed laboratory approach and the students using the traditional method.

The Mann-Whitney U Test was used to compare the attitude scores of the pupils who wrote both the attitude pretest and posttest in the non-directed and control groups. The same test was used to compare the attitude scores of the pupils who wrote only the attitude

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[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]

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posttest in both groups. The results are summarized in Table IX.

TABLE IX

SUMMARY OF ANALYSIS OF DATA COMPARING NON-DIRECTED AND CONTROL GROUPS

Group	N	U	Critical U
Non-Dir. (Pre-Post)	16	69.5	≤ 66
Con. (Pre-Post)	16		
Non-Dir. (Post)	14	104.5	≤ 51
Con. (Post)	15		

Since both parts for testing the null hypothesis 1(c) did not produce significant differences, the null hypothesis was not rejected. In other words, there was no significant difference in the distributions of attitude scores toward mathematics classes between students using the non-directed laboratory approach and the students using the traditional method.

Question 2

Did the type and amount of verbal and written instruction that the pupils received in doing their assignments affect their attitude toward mathematics classes?

Null Hypothesis 2(a):

There is no significant difference in the distributions of attitude scores toward mathematics classes between students using the directed laboratory approach and the students using the non-directed laboratory approach.

The Mann-Whitney U Test was used to test the above hypothesis. The attitude scores of the pupils who wrote both the attitude pretest and posttest in the directed and non-directed groups were compared. Also the attitude scores of the pupils who wrote only the attitude posttest in both groups were compared. The results are summarized in Table X.

TABLE X

SUMMARY OF ANALYSIS OF DATA COMPARING DIRECTED AND NON-DIRECTED GROUPS

Group	N	U	Z	Sig. Level	Critical U
Dir. (Pre-Post)	19	101			≤ 82
Non-Dir. (Pre-Post)	16				
Dir. (Post)	21	112.5	-1.163	.1230	
Non-Dir. (Post)	14				

Since both parts for testing the null hypothesis 2(a) did not produce significant differences, the null hypothesis was not rejected. In other words, there was no significant difference in the distributions of attitude scores toward mathematics classes between students using the directed laboratory approach and the students using the non-directed laboratory approach.

Null Hypothesis 2(b):

> There is no significant difference in the gains of the attitude scores toward mathematics classes within each of the three groups of students.

The Wilcoxon Matched-Pairs Signed-Ranks Test was used to test this hypothesis. Only the attitude scores of the students

in each of the three groups who wrote both the attitude pretest and posttest were used. The results are summarized in Table XI.

TABLE XI

T SCORES AND CRITICAL T SCORES FOR THE THREE GROUPS ON THE ATTITUDE PRE AND POSTTESTS

Group	N	T	Critical T
Dir.	19	50.5	≤ 46
Non-Dir.	16	59.5	≤ 30
Con.	16	27	≤ 30

There was no significant change in the attitude scores of the directed and non-directed groups. There was a significant decrease in attitude posttest scores among the students in the control group who also wrote the attitude pretest.

The analyses of the attitude scores did not produce any significant results. Since there also was an appearance of interaction, one must be cautious in deciding whether the laboratory approach did produce any changes in the students' attitudes toward the mathematics classes. Following is an analyses of student achievement scores in the laboratory and traditional modes.

<u>Analysis of Achievement Scores</u>

The test mean of the 102 students who wrote the achievement test was 15.59 out of a maximum of 30. The test variance was 16.65 while the KR-20 reliability was 0.6718. A histogram of the achievement scores follows.

FREQUENCY	0	0	0	5	7	9	19	21	17	9	8	5	2	0	0
21								*							
20								*							
19							*	*							
18							*	*							
17							*	*	*						
16							*	*	*						
15							*	*	*						
14							*	*	*						
13							*	*	*						
12							*	*	*						
11							*	*	*						
10							*	*	*						
9						*	*	*	*	*					
8						*	*	*	*	*	*				
7					*	*	*	*	*	*	*				
6					*	*	*	*	*	*	*				
5				*	*	*	*	*	*	*	*	*			
4				*	*	*	*	*	*	*	*	*			
3				*	*	*	*	*	*	*	*	*			
2				*	*	*	*	*	*	*	*	*	*		
1				*	*	*	*	*	*	*	*	*	*		
CLASS INTERVAL	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30

Table XII contains the means of the verbal and nonverbal I.Q. scores within the various groups.

TABLE XII

MEANS OF VERBAL AND NONVERBAL I.Q.

SCORES WITHIN VARIOUS GROUPS

Group	Mean of Nonverbal I.Q.	Mean of Verbal I.Q.
All groups	107.7	106.5
Control	104.4	104.7
Experimental	111.0	108.2
Directed	111.9	108.3
Non-Directed	109.9	108.2

Table XIII contains the means, adjusted means and the variances of the achievement scores within the various groups.

TABLE XIII

MEANS, ADJUSTED MEANS, VARIANCES OF

ACHIEVEMENT SCORES WITHIN VARIOUS GROUPS

Group	Mean	Adjusted Mean	Variance
All groups	15.7	15.8	17.8
Control	15.9	16.4	19.4
Experimental	15.4	15.2	16.2
Directed	15.5	15.2	16.1
Non-Directed	15.4	14.9	16.8

Following are the analyses of the achievement scores. All of the following hypotheses were analyzed by using the one-way analysis of covariance with the verbal and nonverbal I.Q. scores as covariates.

Question 3

How were the students' achievements in mathematics affected by the laboratory approach?

Null Hypothesis 3(a):

When the effects of student I.Q. scores are subtracted, there is no significant difference in mean achievement scores between students using the laboratory approach and the students using the traditional method.

Null Hypothesis 3(b):

When the effects of student I.Q. scores are subtracted, there is no significant difference in mean achievement scores between students using the directed approach and the students using the traditional method.

Null Hypothesis 3(c):

When the effects of student I.Q. scores are subtracted, there is no significant difference in mean achievement scores between students using the non-directed approach and the students using the traditional method.

The analysis of testing the above three hypotheses are given in Table XIV, Table XV, and Table XVI respectively.

TABLE XIV

SUMMARY TABLE FOR ANALYSIS OF COVARIANCE RESULTS INVOLVING THE MATHEMATICS ACHIEVEMENT TEST FOR EXPERIMENTAL AND CONTROL GROUPS

Source	DF	MS	ADJ F	P
GRP	1	33.61	2.22	.140
WTH	97	15.17		

TABLE XV

SUMMARY TABLE FOR ANALYSIS OF COVARIANCE RESULTS INVOLVING THE MATHEMATICS ACHIEVEMENT TEST FOR DIRECTED AND CONTROL GROUPS

Source	DF	MS	ADJ F	P
Group	1	21.48	1.31	.256
WTH	67	16.39		

[illegible]

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[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

[illegible]

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

TABLE XVI

SUMMARY TABLE FOR ANALYSIS OF COVARIANCE RESULTS INVOLVING THE MATHEMATICS ACHIEVEMENT TEST FOR NON-DIRECTED AND CONTROL GROUPS

Source	DF	MS	ADJ F	P
Group	1	29.38	1.95	.168
WTH	57	15.09		

The three null hypotheses 3(a), 3(b) and 3(c) were not rejected since the analyses of the achievement scores indicated no significant differences. In other words, from the above results the laboratory approach had no significant effect on student achievement.

Question 4

Did achievement in mathematics depend on the type of laboratory method the student engaged in?

Null Hypothesis 4(a):

> When the effects of student I.Q. scores are subtracted, there is no significant difference in mean achievement scores between students using the directed approach and the students using the non-directed approach.

The above hypothesis was tested by using a one-way analysis of covariance with the verbal and nonverbal I.Q. scores as covariates. The results are summarized in Table XVII.

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[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]

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TABLE XVII

SUMMARY TABLE FOR ANALYSIS OF COVARIANCE RESULTS INVOLVING THE MATHEMATICS ACHIEVEMENT TEST FOR DIRECTED AND NON-DIRECTED GROUPS

Source	DF	MS	ADJ F	P
Group	1	.064	.0043	.948
WTH	66	14.73		

The null hypothesis 4(a) was not rejected. In other words, there was no significant difference in mean achievement scores between students using the directed approach and the students using the non-directed approach.

The results of the analyses of achievement scores provided no significant differences in mean achievement among the three groups.

Teacher Responses (A Case Study)

The last part of this study was concerned with teachers' opinions on teaching Mathematics 15 by the laboratory method.

Question 5

What effect had the laboratory method on teachers' attitudes toward teaching mathematics to low achievers?

Since a small number of teachers, four in total, were involved in using the laboratory approach, no statistical procedures were involved in testing hypothesis 5(a). Two of the

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teachers involved were experienced in teaching Mathematics 15, while the remaining two were teachers who majored in physical education. One of the latter teachers was teaching Mathematics 15 for the first time. Following is a summary of the four teachers' responses to the questionnaire. The teacher questionnaire and the four teachers' responses to each question may be found in Appendix C of this thesis.

The four teachers thought that teaching Mathematics 15 students by the laboratory approach was more enjoyable than by the traditional method. One of the teachers found it much more relaxing because all of the students were quite involved in the laboratory exercises. The teachers' rapport with the students remained the same or was increased.

Three of the teachers found it about the same or easier to teach these pupils in this manner. The remaining teacher found it harder to teach because he had to give more individual or small group help. However, all of the teachers agreed that they did not have to spend more time in preparing lessons.

The teachers thought that the students showed a great interest in this type of approach at the beginning of the experiment but towards the end of the three week experiment the students' enthusiasm decreased. The teachers had mixed feelings about the apparent increase in student attitudes toward learning mathematics by the laboratory approach. Some commented that the increase in student attitudes was due to the novelty effect, while others attributed the increase in attitudes to the challenging and interesting aspects

of the materials.

Three of the four teachers reported that the materials and lessons were not too difficult for the pupils to understand and master. All of the teachers found the materials to be useful and practical in teaching Mathematics 15. Perhaps one of the essential features of this approach was that all of the teachers found it easier to manage and discipline the class.

The teachers agreed that the same results would be obtained with other topics in teaching Mathematics 15 by using the laboratory approach. In general, the teachers felt that all of the Mathematics 15 teaching could be done by using such an approach. As to extending this approach to other mathematics subjects in the high school, the teachers' responses varied. One of the teachers felt that such an approach was best for Mathematics 15 students because they must be active in learning mathematics. Two others felt that such an approach would be beneficial at all levels, while the fourth teacher felt that such an approach would be more beneficial for better students.

The teachers agreed that a major disadvantage of such an approach would be the preparation of the laboratory lessons by themselves. They believed that the time involved in preparing such lessons would be too demanding.

In summary, the four teachers generally favored the laboratory method over the traditional method in teaching Mathematics 15 students. In general they found the students more enthusiastic about mathematics by using such an approach. The major disadvan-

tage would be the time involved if the teachers were to prepare the lessons and materials by themselves.

The present chapter presented the analyses of attitudes and achievements between the following groups: (1) experimental and control; (2) directed and control; (3) non-directed and control; and (4) directed and non-directed. Also the teacher responses to the questionnaire were summarized. In the next chapter the conclusions and implications of the study are presented.

CHAPTER V

CONCLUSIONS AND IMPLICATIONS

The Study

Educators are concerned with the amount of mathematics, both in terms of skills and concepts, being acquired by the low achievers. Apparently, the traditional method of teaching such students does not produce the desired results. Many learning theorists advocate that through the manipulation of concrete materials mathematics will become more realistic and meaningful to the students. Determining the effect of the laboratory approach on the attitudes and achievements of Mathematics 15 students was undertaken in this study. The teachers involved in this project were given an opportunity to express their views on teaching such students by the laboratory method.

The investigators involved in this study wrote two different types of laboratory booklets (directed and non-directed) on a unit in Mathematics 15. This unit involved concepts on counting, combinations, permutations and probability. The emphasis in both of these two different laboratory lessons was on student use of concrete materials in developing concepts. A shoe box containing all the necessary physical materials was provided for the students. The rationale for the development of the two types of laboratory booklets is included in Katherine McLeod's study (1972).

Each laboratory booklet contained two or three different lessons on the same concept. Thus students had several perspectives and different learning experiences for each concept. Also,

this several-lesson structure acted to strengthen the development of the concept for students who could do all of the activities. Optional lessons were developed for enrichment. The optional lessons enabled the teacher to keep students on the same concepts when that was desired. Review questions were provided at the end of each section and also as a final review. The same review questions were given to the control classes to assure parallel coverage. For a sample of some of the laboratory lessons refer to Appendices D and E of this study.

A thorough pilot study was conducted by Katherine McLeod involving twelve Mathematics 15 students at Victoria Composite High School. Any noticeable errors in the prepared materials were corrected by the experimenters. The actual experiment involved six classes of Mathematics 15 students; four classes forming the experimental group and two classes forming the control group. In the experimental classes, half of the students used the non-directed booklet while the other half of the students used the directed booklet. In these classes the students worked in groups of two or three and these groups remained intact for the duration of the experiment (approximately three weeks).

At the beginning of the experiment half of the students, chosen at random in each of the three groups, were given the attitude test. At the conclusion of the three week experimental period all of the students wrote the attitude and achievement tests. Also, at this time, the teachers involved were given a questionnaire. The attitude and achievement tests plus the

the teacher questionnaire and responses are found in Appendices A, B and C respectively.

The following data were collected in this study: (1) attitude pretest scores; (2) attitude posttest scores; (3) achievement scores; (4) student verbal I.Q. scores; and (5) student nonverbal I.Q. scores.

<u>The Attitude Test</u>

The attitude test was administered to assess the changes in attitude relative to the laboratory approach. Comparisons of attitude were made across the two laboratory approaches and the control group. Also, changes in attitude from pre-treatment to post-treatment in each of the groups was assessed.

To test for possible interaction between the pretest and the treatment, a Solomon Four-Group Design was used. The design in symbolic form is as follows:

(1)	R	0_1	X	0_2
(2)	R	0_3		0_4
(3)			X	0_5
(4)	R			0_6

from which the following null hypotheses were tested: (1) $0_2 = 0_1$; (2) $0_2 = 0_4$; (3) $0_5 = 0_6$; and (4) $0_5 = 0_3$. The Wilcoxon Matched Pairs Signed-Ranks Test was used to test the first null hypothesis while the latter three null hypotheses were tested by using the Mann-Whitney U Test. The results of these tests suggested the appearance of interaction between the pretest and the treatment. Null hypothesis (2) was rejected while null

hypotheses (1), (3), and (4) were not rejected.

A possible explanation for the interaction between the pretest and the treatment follows.

Within one of the control classes the posttest attitude scores were lower than those in the other five classes. Therefore the interaction could be attributed to this sampling error.

Comparison of Student Attitudes Between Groups

Students' attitudes between pairs of groups were analyzed using the Solomon Four-Group Design. A summary of the results follow.

The Mann-Whitney U Test was used to compare (1) the attitude posttest scores of the pupils in the experimental and control groups who wrote both the attitude pretest and posttest, and (2) the attitude posttest scores of the pupils in the experimental and control groups who only wrote the attitude posttest. In terms of the Solomon Four-Group Design

(1)	R	0_1	X	0_2
(2)	R	0_3		0_4
(3)	R		X	0_5
(4)	R			0_6

the appropriate null hypotheses were (1) $0_2 = 0_4$, and (2) $0_5 = 0_6$. Both of these comparisons did not produce significant differences.

The Mann-Whitney U Test was used to compare (1) the attitude posttest scores of the pupils in the directed laboratory and control groups who wrote both the attitude pretest and posttest,

and (2) the attitude scores of the pupils in the two groups who wrote only the attitude posttest. In terms of the Solomon Four-Group Design

(1)	R	0_1	directed	0_2
(2)	R	0_3		0_4
(3)	R		directed	0_5
(4)	R			0_6

the null hypotheses were (1) $0_2 = 0_4$, and (2) $0_5 = 0_6$. Both of these comparisons did not produce significant differences.

The Mann-Whitney U Test was used to compare (1) the attitude posttest scores of the pupils in the non-directed and control groups who wrote both the attitude pretest and posttest, and (2) the attitude scores of the pupils in the two groups who wrote only the attitude posttest.

In terms of the Solomon Four-Group Design

(1)	R	0_1	non-directed	0_2
(2)	R	0_3		0_4
(3)	R		non-directed	0_5
(4)	R			0_6

the null hypotheses were (1) $0_2 = 0_4$, and (2) $0_5 = 0_6$. Both of these comparisons did not produce significant differences.

The Mann-Whitney U Test was used to compare (1) the attitude posttest scores of the pupils in the directed and non-directed groups who wrote both the attitude pretest and posttest, and (2) the attitude scores of the pupils in the two groups who wrote only the attitude posttest. In terms of the Solomon Four-Group

Design

(1)	R	0_1	directed	0_2
(2)	R	0_3	non-directed	0_4
(3)	R		directed	0_5
(4)	R		non-directed	0_6

the null hypotheses were (1) $0_2 = 0_4$, and (2) $0_5 = 0_6$. Both of these comparisons did not produce significant differences.

The Wilcoxon Matched-Pairs Signed-Ranks Test was used to test any differences in the gains of the attitude scores toward mathematics classes within each of the three groups of students. In terms of the design

(1)	0_1	directed	0_2
(2)	0_3	non-directed	0_4
(3)	0_5		0_6

the null hypotheses were (1) $0_2 = 0_1$, (2) $0_3 = 0_4$, and (3) $0_5 = 0_6$. The results indicated a significant decrease in attitude posttest scores among the students in the control group who also wrote the attitude pretest. The differences in the other two groups were not significant. As stated before the sampling error related to one of the control classes would seem to explain this.

Student Achievement

The one-way analysis of covariance with the verbal and nonverbal I.Q. scores as covariates was used for making comparisons of the mean achievement scores among the following groups of students:

(1) experimental group versus control group

(2) directed group versus control group

(3) non-directed group versus control group

(4) directed group versus non-directed group.

The results of the above comparisons provided no significant differences in mean achievement among the three groups.

Teachers' Opinions

In summary, the teachers' reactions to teaching Mathematics 15 students by the laboratory method were the following:

(1) In general the teachers favored the laboratory method in teaching such students.

(2) Three of the four teachers found teaching by such an approach to be more enjoyable and relaxing because the students became quite involved in doing the laboratory lessons.

(3) The student-teacher rapport may have been enhanced.

(4) Classes may have been easier to manage and discipline problems may have lessened.

(5) The lessons and materials developed were appropriate for Mathematics 15 students.

(6) A major disadvantage of such a teaching approach is the time factor if the teachers were to prepare the lessons themselves.

For the teachers' responses refer to Appendix C of this thesis.

Summary of Conclusions

There were no significant differences in student post-test attitude scores among the three groups. There was a significant loss in student posttest attitude scores by the students in the control group who also wrote the attitude pretest. The analyses of the student mean achievement scores among the three groups produced no significant results. In general the teachers favored the laboratory approach over the traditional approach in teaching Mathematics 15.

Implications for the Classroom

On the basis of the results in this study there is little evidence that the laboratory approach in teaching Mathematics 15 students improves the students' attitudes and achievement. On the other hand, the results do not suggest that such a teaching approach is inferior in any way to the traditional method. Therefore, it seems suitable for any classroom teacher to use the developed laboratory lessons and materials in teaching Mathematics 15 students as a change from the regular routine. Research results do indicate that variety in instructional approaches does have a positive effect on student achievement.

These laboratory lessons could be used in other mathematics subjects offered in the high schools. For instance, these materials could be used to introduce the probability chapter in Mathematics 30. Many of the items on the achievement test were applicable to the concepts on probability in Mathematics 30.

The length of a class period should not affect the students' performances in these lessons. Frequently the students were in the middle of a lesson at the conclusion of a period and yet were able to continue effectively with the lesson the next day. Also, if a student was absent, his partner worked efficiently and effectively by himself. The absent student's work did not seem to suffer on return. The absent student could master the concept by doing the other laboratory lessons involving the same concept.

One of the teachers in this study was inexperienced and yet he did enjoy using such an approach. Therefore, teachers should not be hesitant in using the laboratory approach because they are inexperienced in teaching mathematics.

To use these materials, a teacher must duplicate the laboratory lessons and make up the concrete materials. Each set of concrete materials costs approximately two dollars. The writer strongly recommends the use of these materials as a change from the regular routine. Teachers may easily modify the lessons as they see fit.

Implications for Further Research

Much research is needed in the use of the laboratory approach as compared to other methods in teaching mathematics. The writer envisions that because of this study researchers can select many experiments of interest and value. The following are a few of the possibilities:

(1) Studies similar to this one to be conducted with more classes to have a better representation of the population. This would include urban as well as rural students.

(2) The duration of the experiment could be varied. Conduct a similar experiment in unsemestered schools where the length of a period is only 40 minutes.

(3) One may wish to conduct a similar experimental incorporating two or three chapters in Mathematics 15.

(4) Conduct a similar experiment in the fall semester when the cold weather does not influence the students. After Easter the truancy rate is high for low achievers.

(5) Conduct a similar experiment where one sets up definite guidelines to differentiate between the two types of laboratory lessons.

(6) Conduct an experiment to determine what type of student aptitudes and characteristics best conform to learning mathematics by the laboratory approach.

(7) Conduct a similar experiment in the following areas:

(a) junior high option classes

(b) other high school mathematics subjects

(c) special schools where the entrance requirements are based on a student's I.Q. and previous achievement

(d) repeat with the same teachers to assess the importance of experience.

In summary it may be wise to incorporate the laboratory approach in the junior high and elementary schools where the

attitudes and habits are initially formed. From the research of literature, students in junior high and elementary schools enjoy working with concrete materials and may find mathematics enjoyable, meaningful, and successful.

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APPENDIX A

ATTITUDE TEST

APPENDIX A

QUESTIONNAIRE

At the top of the next page to be given to you, you will notice the statement:

Learning and Doing Math. 15

Below this statement are a series of word pairs. For example:

happy ____, ____, ____, ____, ______ sad

You are to react to this statement by placing an "X" in one of the five spaces separating the two paired words. Mark your "X" in the space that best indicates the degree of your feeling toward the statement expressed by the word pairs.

For example:

(a) If your feelings toward Math 15 classes suggest to you very strongly the idea "happy" then you would mark it as follows:

happy __X__, ____, ____, ____, ____ sad

(b) If your feelings toward Math 15 classes suggest to you a slight strong feeling of the idea "happy" then you would mark it as follows:

happy ____, __X__, ____, ____, ____ sad

(c) If your feelings toward Math 15 classes suggest to you a feeling which is not happy and not sad (neutral) then you would mark it as follows:

happy ____, ____, __X__, ____, ____ sad

(e) If your feelings toward Math 15 classes suggest to you very strongly the idea "sad" then you would mark it

as follows:

happy _____, _____, _____, _____, __X__ sad

If your feelings are neutral about a word pair, or if you feel the word pair is unrelated to your feelings, place the "X" in the middle space.

<u>IMPORTANT</u>: 1. Be sure to mark only one "X" in the spaces for every word pair. DO NOT OMIT any word pairs.

2. Mark down your first feelings when you read the word pairs. We want your true feelings, however do not be careless.

LEARNING AND DOING MATH. 15

1. worthwhile	_____	_____	_____	_____	_____	worthless
2. not secure	_____	_____	_____	_____	_____	secure
3. familiar	_____	_____	_____	_____	_____	strange
4. happy	_____	_____	_____	_____	_____	sad
5. deadly	_____	_____	_____	_____	_____	lively
6. hard	_____	_____	_____	_____	_____	easy
7. failure	_____	_____	_____	_____	_____	successful
8. student centered	_____	_____	_____	_____	_____	teacher centered
9. like	_____	_____	_____	_____	_____	dislike
10. practical	_____	_____	_____	_____	_____	not practical
11. not creative	_____	_____	_____	_____	_____	creative
12. useless	_____	_____	_____	_____	_____	valuable
13. fair	_____	_____	_____	_____	_____	unfair
14. stupid	_____	_____	_____	_____	_____	smart
15. fast	_____	_____	_____	_____	_____	slow
16. unsure	_____	_____	_____	_____	_____	sure
17. dull	_____	_____	_____	_____	_____	interesting
18. unimportant	_____	_____	_____	_____	_____	important
19. tense	_____	_____	_____	_____	_____	relaxed
20. bad	_____	_____	_____	_____	_____	good
21. active	_____	_____	_____	_____	_____	inactive
22. enjoyable	_____	_____	_____	_____	_____	dull
23. favorite	_____	_____	_____	_____	_____	least favorite
24. different	_____	_____	_____	_____	_____	usual
25. boring	_____	_____	_____	_____	_____	interesting

APPENDIX B

ACHIEVEMENT TEST

APPENDIX B

ACHIEVEMENT TEST

TEST - COUNTING, ORDERING, COMBINATIONS & PROBABILITY

Circle your choice.

1. Set A contains 7 elements. The total number of possible orders by taking 4 elements at a time is:

   A. 7 x 6 x 5 x 4 C. 4 x 4 x 4 x 4

   B. 7 x 6 x 5 x 4 x 3 x 2 x 1 D. 7 x 7 x 7 x 7

2. The Jones couple wishes to name their son so that his initials are ACJ. If they choose from Alvin, Alexander, Andrew, Alfonse, Chuck, Curtis, Colin, in how many ways might the son be named?

   A. 1 B. 3 C. 4 D. 12

3. In how many ways can 5 different colored marbles be arranged in a row?

   A. 5 B. 3125 C. 120 D. 15

4. A penny is tossed 3 times. Find the probability of obtaining 2 heads and 1 tail.

   A. 1/8 B. 3/8 C. 1/4 D. 1/2

5. A bag contains 2 white, 4 blue and 6 red marbles. If 1 marble is drawn from the bag, what is the probability that it is NOT blue?

   A. 2/3 B. 1/3 C. 1/8 D. 1/4

6. There are 4 redheads in a group of 30 people. The probability of selecting at random 2 redheads from the group is:

   A. $\frac{4}{30} + \frac{3}{29}$ B. $1 - \frac{2(26)}{2(30)}$ C. $\frac{4}{30} \times \frac{2}{26}$ D. $\frac{4}{30} \times \frac{3}{29}$

7. Two dice - 1 red, 1 green - are tossed. If the red die turns up 6, what is the probability of obtaining a total of 11 as the sum of the dots which show up on the two dice?

   A. 1/18 B. 1/2 C. 1/6 D. 2/3

8. There are 4 green, 3 white, and 5 red marbles in a bag. The probability that 2 marbles drawn (first marble is not replaced) from the bag will <u>BOTH</u> be white is:

A. 1/11 B. 19/44 C. 1/24 D. 1/22

9. Find the probability that 2 heads will turn up when 2 coins are tossed is:

A. 3/4 B. 1/2 C. 1/4 D. 1

10. Three different colored dice are thrown at once. How many different outcomes are possible?

A. 1 B. 216 C. 6 D. 36

11. In how many ways can 8 different books be arranged on a shelf?

A. $\frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{2}$ C. 8

B. 7 x 6 x 5 x 4 x 3 x 2 x 1 D. 8 x 7 x 6 x 5 x 4 x 3 x 2 x 1

12. What is the probability of throwing two dice so that the sum of the dots facing up is nine?

A. 1/9 B. 1/4 C. 1/3 D. 1/36

13. How many different committees of 3 persons can be formed in a club having 6 members?

A. 120 B. 20 C. 6 D. 720

14. A person throws 3 different coins at once. How many of the possible outcomes will contain exactly 2 heads?

A. 8 B. 2 C. 3 D. 6

15. There are four different posters available for an arrangement in a row. The number of different arrangements that can be made if 2 posters are used is:

A. 60 B. 24 C. 12 D. 48

16. A die is rolled and a card is drawn from a deck of 52 playing cards. The probability of getting a 6 and a <u>BLACK</u> king is:

A. 1/6 x 2/52 B. 1/6 x 1/52 C. 1/6 x 2/51 D. 1/6 + 2/52

17. A card is drawn at random from an ordinary deck of 52 cards. What is the probability that the card is green?

A. 1/13 B. 1/4 C. 1/52 D. 0

18. A bag contains 3 black balls, 2 white balls and 1 red ball. Two balls are drawn from the bag. If the first ball is replaced before the second ball is drawn, find the probability that both balls are black.

A. 1/2 B. 1/4 C. 1/9 D. 1/36

19. A team has 10 players. In how many ways can 2 players be selected as co-captains?

A. 10 x 9 x 8 x 7 x 6 x 5 x 4 x 3 x 2 x 1 C. 45

B. 90 D. 2

20. A regular hexahedron has 6 faces. One of its faces is painted white, 2 are painted red, and the rest are painted blue. If the hexahedron is rolled onto a flat surface, what is the probability that it will come to rest on a blue face or red face?

A. 5/6 B. 1/3 C. 1/2 D. 1/6

21. The probability of getting a sum of 8 on a toss of 2 dice is:

A. 1/22 B. 1/2 C. 1/12 D. 5/36

22. A town issues license plates for bicycles. Each plate has one of the 26 letters of the alphabet followed by one of the 10 digits. How many different plates are possible?

A. 26 B. 10 C. 26 x 10 D. 26 + 10

23. Three coins are tossed into the air. The probability that none lands heads up is:

A. 3/8 B. 1/8 C. 0 D. 1/2

24. For a family of three children, if each child is just as likely to be a boy as a girl, what is the probability that all three children are girls?

A. 1/2 B. 1/4 C. 3/4 D. 1/8

25. All of the possible different outcomes for flipping a quarter and a penny is:

A. (H,T) B. (H,H), (H,T), (T,H), (T,T)

C. (H,H), (H,T), (T,T) D. (H,T), (T,H), (T,T)

26. A bag contains 6 red balls and 3 blue balls. If a die is thrown, what is the probability of getting 3 or less on the die OR picking a blue marble from the bag?

A. 1/6 x 3/9 B. 3/6 x 3/9 C. 1/6 + 3/9 D. 3/6 + 3/9

27. A luncheon menu allows you to choose a soup from 4 different soups and a sandwich from 6 different sandwiches. How many different lunches can you have?

A. 4 B. 6 C. 24 D. 10

28. All radio stations in Edmonton are named by 4 letters. The first letter is "C". How many radio stations can be obtained by filling in the blanks in "C - - -" with three letters to be chosen from the remaining 25 letters, without repetition?

A. 13800 B. 26 C. 75 D. 15625

29. A card is drawn at random from an ordinary deck of 52 cards. What is the probability that the card is a heart?

A. 1/52 B. 1/4 C. 1/2 D. 1/3

30. Jane rolled a die 7 times. Each time a 3 on the die showed up. What is the probability of Jane rolling a 3 on the next toss of the die?

A. 1/6 B. 5/6 C. 3/6 D. 1/7

APPENDIX C

TEACHER QUESTIONNAIRE

AND

THE TEACHERS' RESPONSES

APPENDIX C

TEACHER QUESTIONNAIRE AND THE TEACHERS' RESPONSES

1. In presenting the concepts in this manner, is teaching more enjoyable than traditional method?

   Teacher A. "Much more relaxing as all of the students get quite involved in their labs."

   Teacher B. "Yes."

   Teacher C. "Yes, for Math 15."

   Teacher D. "In the sense that students at different achievement levels do not affect the class as much."

2. Is rapport with students better?

   Teacher A. "In some instances -- although most were able to work on their own."

   Teacher B. "About the same!"

   Teacher C. "Yes, for Math 15."

   Teacher D. "The same students dislike you regardless of method."

3. Is it harder to teach in this manner?

   Teacher A. "No -- not at all."

   Teacher B. "No."

   Teacher C. "No."

   Teacher D. "You are helping smaller groups, because they are all at different places in the experiment. In this sense it is harder."

4. Do you have to spend more time in preparing your lesson?

   Teacher A. "No -- about the same."

   Teacher B. "No."

   Teacher C. "No."

   Teacher D. "You would have to spend much more time to organize a course like this."

5. Did you find there was more, or less, interest shown by students in this method of presenting concepts?

   Teacher A. "More -- but possibly because of the change from the traditional method of teaching."

   Teacher B. "More interest at first."

   Teacher C. "Greater interest at first; from then it was up and down depending on lesson."

   Teacher D. "Greater interest was shown in the experiments, but the concepts?"

6. Comment on students' attitudes.
   If better attitudes of students, was it due to novelty of doing something different from other classes, or was it due to students finding materials interesting and challenging?

   Teacher A. "The attitude was pretty well the same -- but their concentration span did appear to be a bit better."

   Teacher B. "For some students in the class it was the challenging aspect of the work."

   Teacher C. "Mixture of both."

   Teacher D. "My personal opinion is the interest and attitude improvement was the novelty."

7. Did students find materials (lessons) too hard?

   Teacher A. "No -- they were quite well written."

   Teacher B. "No. Even the poor students could play the games and make deductions. Right or wrong they were thinking, some perhaps for the first time. This is perhaps the greatest feature of this approach."

   Teacher C. "No."

   Teacher D. "In most instances -- yes."

8. Is such an approach to teaching useful and practical?

   Teacher A. "Yes -- I feel so."

   Teacher B. "It is practical for the teacher and useful because the students have more freedom and the teacher does not have to motivate or 'club' the students to do something."

Teacher C. "Yes."

Teacher D. "Yes, very much so."

9. Is it harder to manage or discipline the class by using the lab approach?

Teacher A. "Much easier -- as once they can get involved they continue on the lab."

Teacher B. "No."

Teacher C. "No."

Teacher D. "No -- they work more conscientiously."

10. Do you feel that the results would be the same for other topics?

Teacher A. "Possibly -- cannot really answer."

Teacher B. "Depending upon the type of lab that was set up the results should be similar."

Teacher C. "Yes."

Teacher D. "Yes."

11. Do you feel results would be the same for other types of students? Is it particularly appropriate for the Math 15 students?

Teacher A. "Excellent for Math 15 -- would be good also for other types of students."

Teacher B. "Math 15 people need something to do rather than listen. They must be active! From this point of view the approach is ideal."

Teacher C. "More appropriate for Math 15. Would work at all levels though."

Teacher D. "I think you would have better results with concepts with better students."

12. If you feel this method is quite worthwhile, would you be willing to make up your own labs?

Teacher A. "Yes -- on a smaller scale."

Teacher B. "I have made up labs of my own previous, but it is impossible to keep ahead because it takes too much time to prepare them properly."

Teacher C. "Yes, except time is a factor."

Teacher D. "It would take more time than a teacher teaching 7 periods could find time to complete in one term."

13. Do you think this approach could be used for all of Math. 15?

Teacher A. "Pretty well all."

Teacher B. "It could be used, but I think it would lose its effectiveness."

Teacher C. "Yes."

Teacher D. "Yes. I think it has greater merit than other approaches I have tried."

14. Do you feel that the preparation of your own lab materials would be too time-consuming, even though you feel that the lab approach is worthwhile?

Teacher A. "Possibly -- but only for the writing and typing time."

Teacher B. "Yes! because unless the labs are carefully prepared, very little learning will take place."

Teacher C. "Yes."

Teacher D. "Yes."

15. For what type of Math. 15 student do you feel the lab approach benefits the most? (i.e. those who show initiative, those who are slow, the above-average, etc.)

Teacher A. "I feel this is best for the slightly below and average student as the below average cannot comprehend and the above average find these particular labs not very demanding."

Teacher B. "The student who is basically concerned about learning even though he may have previously not shown any interest. If he wanted to learn, then here was a way of learning without the usual peer pressure against it."

Teacher C. "Labs of varying difficulty could be used for all levels and kinds."

Teacher D. "Those who show initiative."

16. Additional Comments -

Teacher A. None.

Teacher B. "On too many occasions the students were not looking or finding the insight into the problems they should have. They might if they were used to try to delve into problems rather than being used to the superficial analysis they usually apply."

Teacher C. None.

Teacher D. "I am still not convinced that it helps some students to know why they use a certain procedure to solve mathematical problems. Math 15 seem content to be shown and seem to only be able to operate in this manner."

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

APPENDIX D

SAMPLES OF DIRECTED LABORATORY LESSONS

SAMPLE LESSON 1

PICK YOUR CHOICE

MATERIALS: 2 different colored dice, and a pink and blue gameboard.

1. When throwing two dice do you have a better chance of winning with a sum of 6, 7, or 8 than with a sum of 2, 3, 4, 5, 9, 10, 11, 12? Why?

2. Perhaps the following table will help you answer the above question. Complete the table. The first number stands for the outcome on the green die and the second number stands for the outcome on the red die.

TABLE A

		Die 1 (red)					
		1	2	3	4	5	6
Die 2 (Green)	1						
	2						
	3						
	4						
	5						
	6						

Remember that an outcome of 2 on the green die and 5 on the red die (we'll write it as (2,5)) is different from an outcome of 5 on the green die and 2 on the red die (we'll write this as (5,2)).

(a) How many ways can you throw a sum of 8?__________
List them. __________

(b) How many ways can you throw a sum of 7? __________
List them __________

(c) How many ways can you throw a sum of 6?__________
List them __________

(d) Answer #1 again. __________

PICK YOUR CHOICE

Two players are to play this game.

1. There are 15 squares, some are blue and some are pink on large card-board.

2. Flip a coin. The winner chooses the color he prefers and will toss the 2 dice while the other person fills in Table 1.

3. Toss 2 dice. Make the same number of moves in the diagram as the sum of the dots on the two dice.

4. If you end on a pink - give pink one point.
   If you end on a blue - give blue one point.

5. On each toss of the dice, always start counting at "start".

6. Toss the dice 20 times (1 game).

7. Repeat this 4 times (or play 4 games).

TABLE 1

	Tally for Pink	Tally for Blue	Score	
			Pink	Blue
Game 1				
Game 2				
Game 3				
Game 4				

1. Do you think this game is fair? ______________ Why? _____________

   ____

   ____

2. (a) How many squares are there on the game board? _______________

   (b) Do you need this many? _______ Why or why not? _____________

   ____

   ____

3. Complete Table 2 on the following page. Hint: Use the table you made on the first page.

TABLE 2

Sum of Dice	List Possibilities	No. of Outcomes
1	Two dice can't add up to 1	0
2	(1,1)	1
3		
4	(1,3) (2,2) (3,1)	3
5		
6		
7		
8		
9		
10		
11		
12		

4. Refer to Table 2.

   (a) In tossing two dice, which sum has the BEST chance of appearing?

   ____

   (b) In tossing two dice, which 2 sums have the LEAST chance of appearing? ______________________________

5. (a) Will a sum of 7 land you on the blue? ____________________

   (b) Will a sum of 2 or 12 land you on pink? ____________________

6. How many different ways can you throw two dice? ____________________

7. Out of the 36 possible outcomes of throwing 2 dice how many of these will yield a point for the blue? (Shade Table "A")

   ____

8. How many outcomes will yield a point for pink? ____________________

   ____

   There are twice as many possible outcomes for the blue as there are for the pink.

9. Again, is this a fair game? Explain.______________________________

   ____

   ____

SAMPLE LESSON 2

JIM'S DILEMMA

MATERIALS: chips of 4 colors, or chips of 3 colors plus cards.

Problem: In how many ways can you arrange 3 different colored poker chips? You have enough chips so that you do not have to destroy any of your previous arrangements.

1. In how many ways can the first chip be chosen?

2. If you choose a red one, how many choices do you have for choosing the second chip?

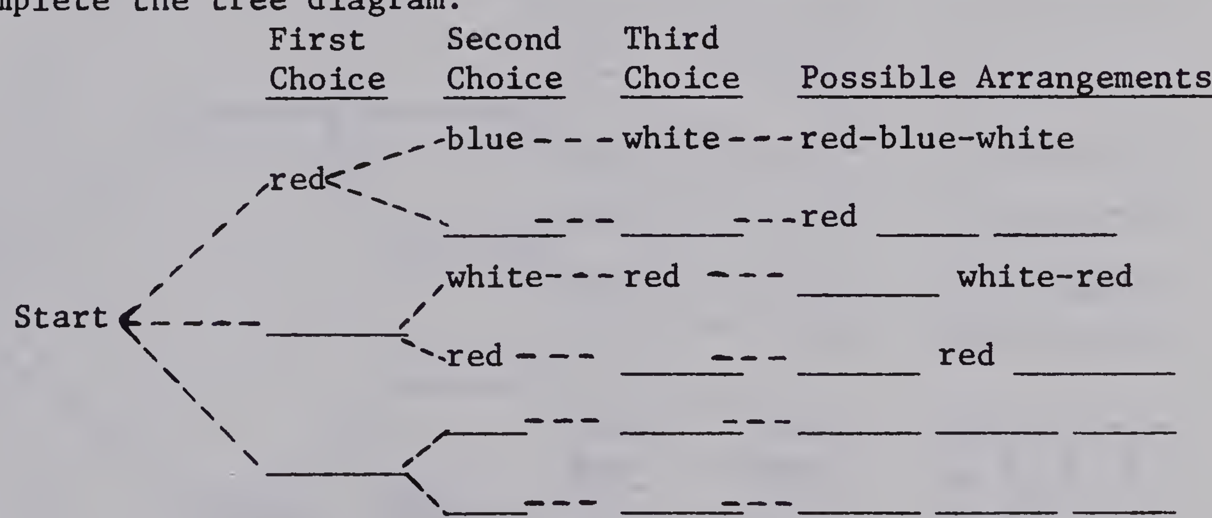
3. If your second choice is the blue chip, how many chips do you have left for your last choice?

4. Certainly your first pick does not have to be the red chip, the second choice the blue chip and the third choice the white chip. This is only one way of arranging 3 different objects.

   (a) In how many ways can you arrange these 3 objects?

   (b) List these arrangements.

5. Now complete the tree diagram.



3 choices x 2 choices x 1 choice = 6 total choices.

Problem: In how many ways can you arrange 4 different objects (3 different colored poker chips and a card)? You should have enough material so that you do not have to destroy any of your arrangements.

6. In how many ways can the first object be chosen?

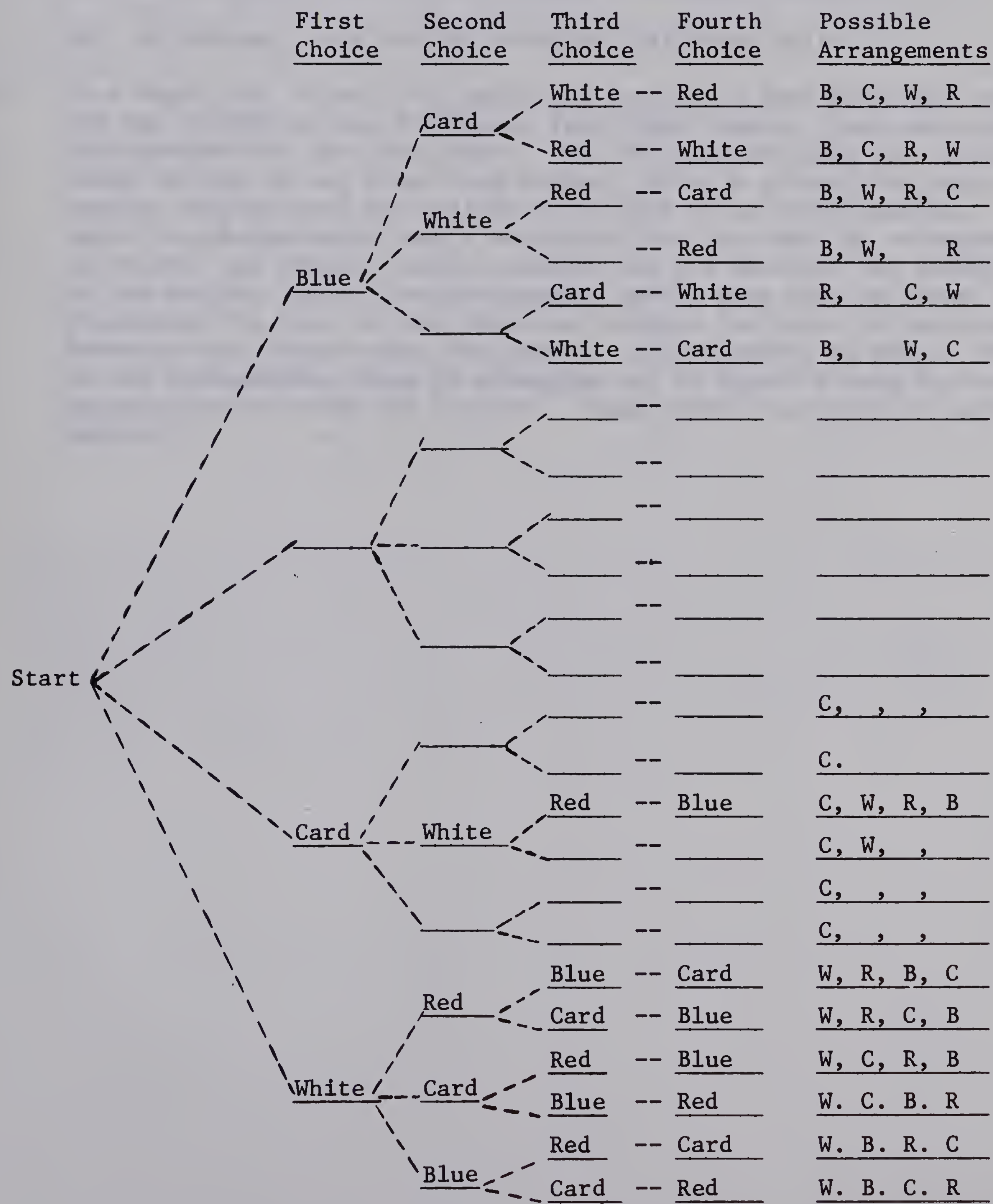
7. If you choose a blue chip, how many objects are left for your second choice?

8. If your second choice is the card, how many objects are left for your third choice?

9. If your third choice is a white chip, how many objects are left for your fourth (last) choice?

10. In how many ways can you arrange (order) the 4 different objects?

11. Now complete the following tree diagram:



________ choices x ________ choices x 2 choices x ________ choices =

24 arrangements

[illegible]

[illegible]

12. You have noticed that the number of arrangements of

    (a) 3 different objects is 3 x 2 x 1 or 3! (read as 3 factorial) = 6

    (b) 4 different objects is 4 x 3 x 2 x 1 or 4! (read as 4 factorial) = 24

    (c) In how many ways can you arrange 2 different objects?

    (d) In how many ways can you arrange 5 different objects?

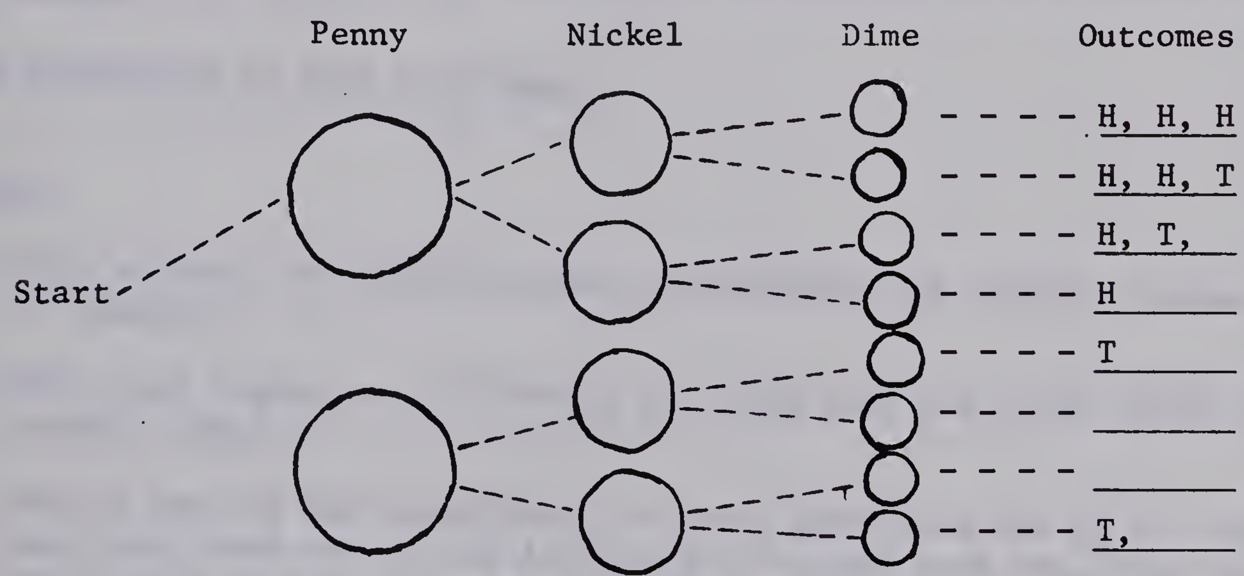
13. Five boys, Jim, Peter, Pat, Harry and Bruce from Jack Pike High School are the winners at the Provincial Tract Meet Finals. They are to be photographed for the local paper. The photographer told the boys to stand in line in any order they wished. After a minute, the photographer decided that the tallest boy should stand in the middle. Again the photographer wasn't satisfied that this was the arrangement he wanted, and after a minute decided that the shortest boy should be in the middle. Again the photographer wasn't sure that he wanted to photograph the boys in this order and changed the order of the boys. Remember each change takes one minute. Jim whispered to Bruce, "Gosh, if the photographer keeps re-arranging us, we might be here for two hours before he takes the picture". Could Jim's statement be correct? Explain!

SAMPLE LESSON 3

GRASSHOPPER GAME

MATERIALS: 3 different coins; penny, nickel, dime.

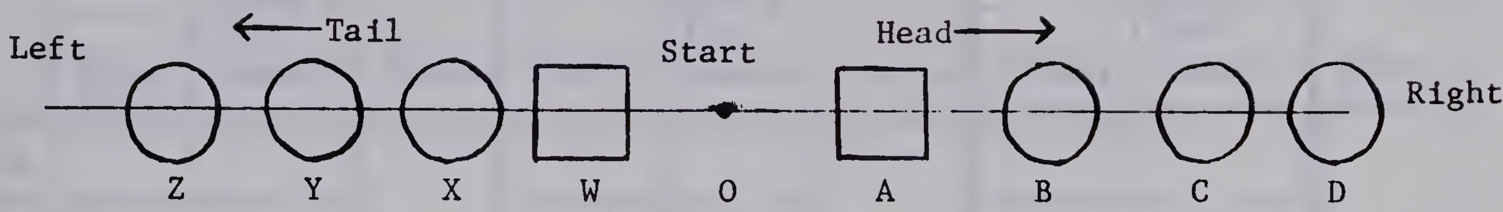
1. What are the possible outcomes when 3 coins are tossed at a time, penny, nickel, dime)? Below is a tree diagram which is partially completed. Fill in the rest. If you are having problems, toss the three coins and notice the possible outcomes.



2. The fraction of heads on penny, tails on nickel, tails on dime, or (H,T,T) to the total number of possible outcomes is 1/8. Ratio of number of (H, H, H) to total number of outcomes possible is __________. Is the ratio of number of (H, H, T) to number of possible outcomes _______________. Is the ratio of number of (H, H, T) to the number of possible outcomes the SAME as the ratio of the number of 2 heads and a tail (in any order) to the total number of possible outcomes? Why not?

GRASSHOPPER GAME

MATERIALS: 3 different coins: penny, nickel, dime



The circles and squares are represented by letters at the bottom of these positions.

Two People are to Play This Game.

Rules:

1. Flip a coin, the winner chooses the position he prefers (either circle or square).

2. Each play begins at "O" (START) and then toss the three coins (penny, nickel, dime).

3. Notice the way the coins land, for each head move one to the right, for each tail move one to the left (i.e. if coins land THH, your moves would be O to W to O to A and therefore one point for squares. If on next toss coins landed HHH, your moves would be O to A to B to C and a point for the circles.

4. One person is to flip the coins, the other is to record the results in Table 1.

5. 16 plays (16 tosses of 3 coins) make up a game.

6. Play 3 games.

7. After each game, total your scores. The one with the most tallies wins.

8. NOTE: Frequency means the number of times something occurs.

TABLE 1

	GAME 1			GAME 2			GAME 3		
	Tally	Fre- quency	Score	Tally	Fre- quency	Score	Tally	Fre- quency	Score
A									
W									
Z									
Y									
X									
B									
C									
D									

TABLE 2 (For all 3 Games)

Place Play Ends	Frequency (Total for 3 Games	Fraction of Total Plays
A		
W		
Z		
Y		
X		
B		
C		
D		

All the Fractions Add up to 1.

Total =

The Fractions should add up to 1.

1. Do you think this game is fair? Why?

2. If you played this game again, what shape would you choose? Why?

3. Use Table 2 to predict at what places a play is most likely to end. Why?

4. Why is the frequency for each of the positions Z, X, B, D = 0? Explain.

5. How many possible outcomes for tossing 3 coins? (refer to tree diagram).

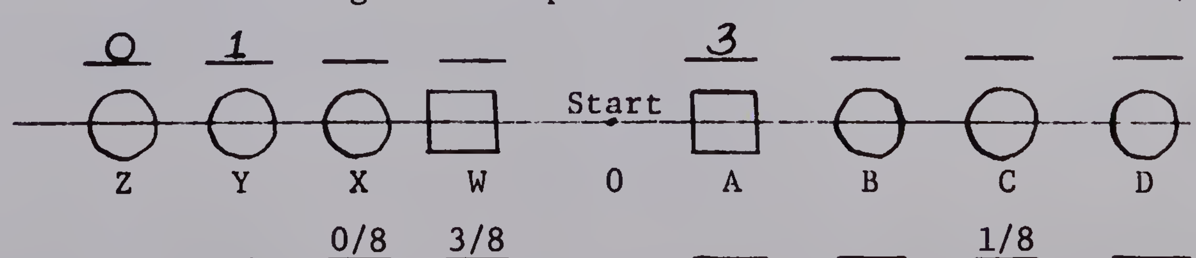
6. Use the tree diagram to complete Table 3.

TABLE 3

Place Outcome Ends	No. of Outcomes Ending at each place	Fraction of total outcomes
A		3/8
B	0	
C		
D	0	0/8
X		
Y		
Z	0	0/8
W		
Total	8	8/8 = 1

7. There are 8 possible outcomes of tossing three coins. Fill in the following: (with the aid of Table 3 and the tree diagram).

No. of outcomes ending at each place



Fraction of outcomes ending at each place

8. Are your fractions in Table 2 the same as in Table 3? Explain any differences.

9. What you have been calculating are ratios or <u>PROBABILITIES.</u>

   Probability may be defined as the $\frac{\text{no. of favorable outcomes}}{\text{total No. of possible outcomes}}$

   which is what you have been doing. Why is there a greater possibility of landing at W or A than at any of the other positions?

10. For 128 trials in the Grasshopper game, using your probabilities from Table 3, how many trials would end at

    (a) W __________ (e) A __________

    (b) X __________ (f) B __________

    (c) Y __________ (g) C __________

    (d) Z __________ (h) D __________

    Did you get 48, 0, 16, 0, 48, 0, 16, 0? Do these add up to 128? What conclusions can you make?

11. How does the total of your scores (from Table 2) in the ratio of the number of squares to the number of circles compare to the calculated probabilities of

    $\frac{A + W}{Y + C}$? If these ratios differ, give an explanation.

12. How would you make this game fair?

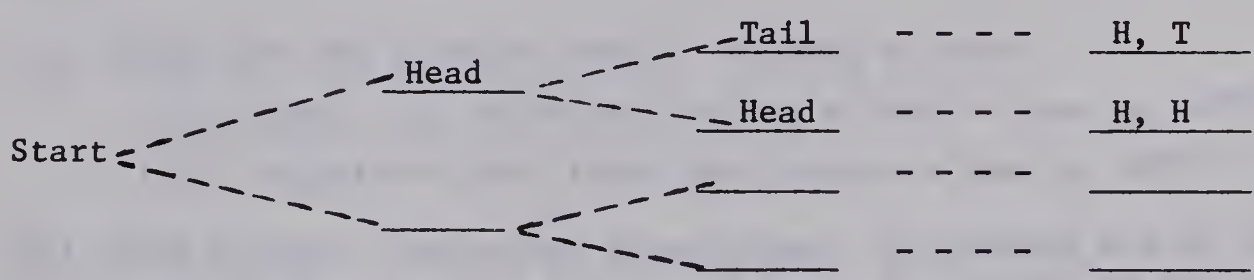
SAMPLE LESSON 4

P Q (Die Vs Coin)

MATERIALS: A die and a coin

1. Below is a tree diagram of tossing a single coin twice. Complete it.

First Toss Second Toss Possible Outcomes

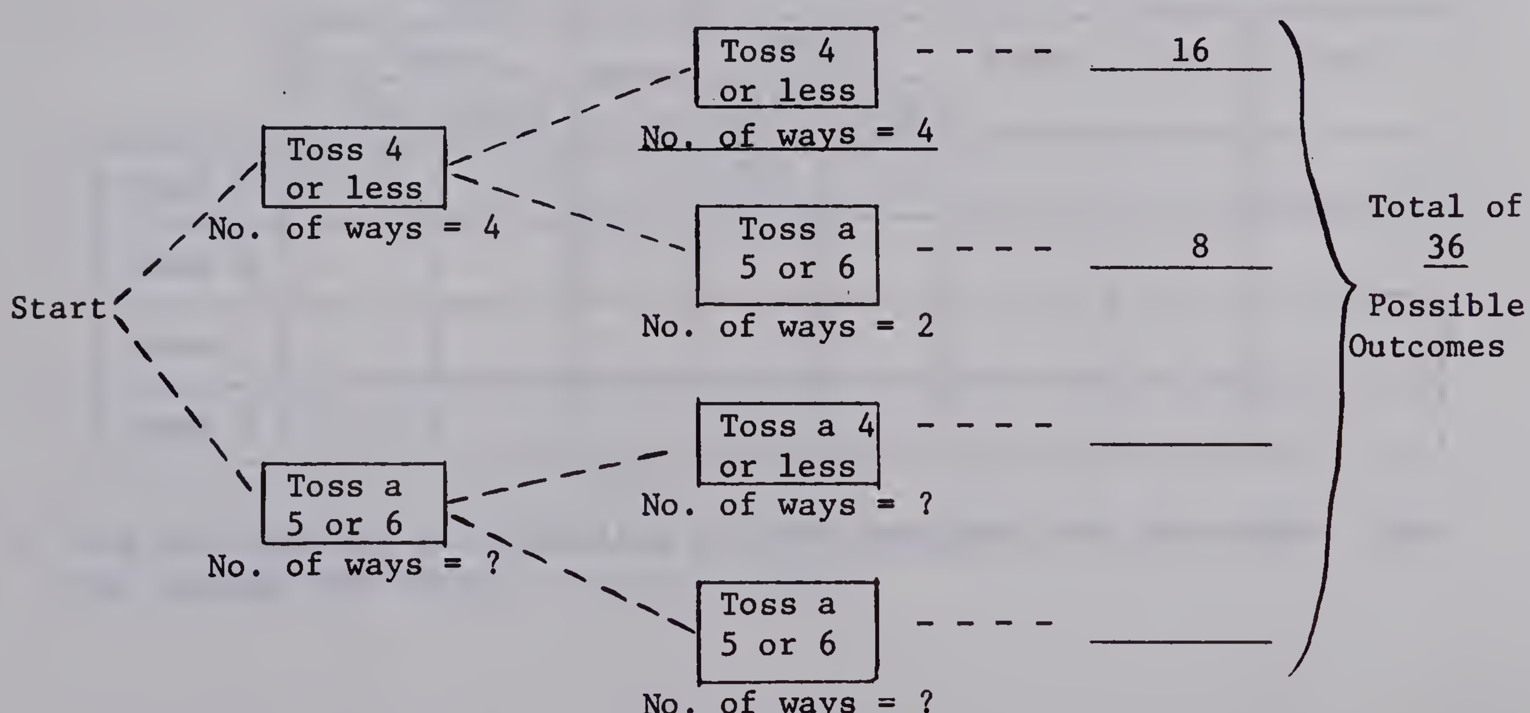


2. Out of the 4 possible outcomes, what fraction of these leads to getting 2 heads in a row?

3. If you toss a die twice, how many possible outcomes are there?

4. Below is a tree diagram indicating the number of ways one may obtain 4 or less or: a 5 or 6, on 2 tosses of a die.

First Toss Second Toss Total Number of Ways



[illegible]

5. Out of the total of 36 possible outcomes, what fraction of these will lead to getting a 4 or less on each of the 2 tosses of a die?

6. Now play the following game in which 2 people are to participate:

Rules:

(a) There are two possible ways of scoring a point:

(i) Toss a die twice and getting a four or less on BOTH tosses,

(ii) Flipping a coin twice and getting a head on BOTH flips.

(b) Flip a coin. The winner plays first. He chooses one of the two routes as outlined in 6 (a). If he is successful on any try, he scores a point. He is to repeat this 10 times.

(c) The other player now chooses the other route and repeats it 10 times.

(d) When both players have had 10 chances, this will make up one game.

(e) Play 4 games.

(f) Fill in Table 1.

TABLE 1

	Person 1			Person 2			Winner	
	Route		Score	Route		Score	Die	Coin
	Die	Coin		Die	Coin			
Game 1								
Game 2								
Game 3								
Game 4								

7. Did you find one route leading to more successes than the other? Can you account for this?

8. From the tree diagram in question 1, there were ________possible outcomes. How many of the outcomes were H, H? ________
Therefore the ratio of

$$\frac{\text{number of favorable outcomes}}{\text{total number of possible outcomes}} = \_\_\_\_\_\_\_\_$$

9. Let's do this without a tree diagram. If you toss a coin once, there are ________possible outcomes. Only one of these is heads. Therefore the probability of obtaining heads on first flip is ________. Again on second flip there are 2 possible outcomes. One of these is a head. Therefore the probability of obtaining a head on the second flip is __________.

The probability of obtaining 2 heads in a row is:

$\frac{1}{2}$ (multiply, add, subtract, divide) $\frac{1}{2} = \frac{1}{4}$

You must multiply the probabilities of the separate events because BOTH events must occur.

10. By referring to question four, how many possible outcomes are there for tossing a die once?

11. Out of the 6 possible ways, how many of these give an outcome of 4 or less?

12. What is the probability of obtaining a 4 or less on the first toss of a die?

13. For the second toss of the die, there are 6 possible outcomes and of these four are favorable. The probability of getting a 4 or less on the second toss is ____________.

14. The probability of obtaining 4 or less on first toss = 4/6 = 2/3.
The probability of obtaining 4 or less on second toss = 2/3.
Are you going to (multiply, add, subtract, or divide) the 2 probabilities above? Why?

15. You have noticed that the probability of obtaining a 4 or less on both tosses of a die is 2/3 x 2/3 = _________.

16. The probability of getting 2 heads in a row = 1/4.
The probability of getting a 4 or less in 2 tosses of a die = 4/9. Which one has a better chance of winning? Why?

17. Did the die in Table 1 have more points than the coin? If not, try to explain why this happened.

18. Which of the following would you choose?

    (a) Toss a die twice, must get a 4 or less on both tosses.

    (b) Toss a die three times, must get a 5 or less on all three tosses.

    Take a die and toss it twice - repeat 20 times - how many times did you have a success?__________ Then toss the die 3 times - repeat 20 times - how many successes? __________

    Calculate the probability for (a) and (b). Did you get 4/9 and 125/216? Which probability is larger?

SAMPLE LESSON 5

6 OR UNDER, 7 OR OVER

MATERIALS: 2 different colored dice

1. How many different outcomes are there in tossing 2 dice? Complete the following partially completed tree diagram. Use the dice to help you if necessary.

	Red Die	Green Die	Possible Outcomes	Sum
		1	(1,1)	2
		2	(1,2)	3
	1	3	(1,3)	4
		4	(1,4)	5
		5	(1,5)	6
		6	(1,6)	7
		___	(2,)	___
		___	(2,)	___
	2	___	(2,)	___
		___	(2,)	___
		___	(2,)	___
		___	(2,)	___
		1	(3,1)	4
		2	(3,2)	5
Start	3	3	(3,3)	6
		4	(3,4)	7
		5	(3,5)	8
		6	(3,6)	9
		___	(4,)	___
		___	(4,)	___
	4	___	(4,)	___
		___	(4,)	___
		___	(4,)	___
		___	(4,)	___
		___		___
		___		___
		___		___
	5	___		___
		___		___
		___	(5,6)	11
		1	(,1)	___
		2	(,2)	___
	6	3	(,3)	___
		4	(,4)	___
		5	(,5)	___
		6	(,6)	___

___ choices x 6 choices = 36 possible outcomes

[illegible]

[illegible]

6 OR UNDER, 7 OR OVER

MATERIALS: 2 different colored dice (red & green)

Rules: Two Players are to play the following game.

1. Toss two dice 20 times. This will make up a game.
2. Notice the sum of the 2 dice on EACH toss.
3. There are 2 possible ways of scoring a point.

   (a) 6 or Under - if the dots on the two dice add up to 6 or less, score a point for the person who chose this situation.

   (b) 7 or over - if the dots on the two dice add up to 7 or over, score a point for the person who chose this situation.

4. Flip a coin. The winner chooses the situation for scoring a point he prefers as outlined in number 3 (above). The other person gets the other way of scoring points.
5. Play 4 games.
6. Fill in Table 1 as you play each game.

TABLE 1

	6 or Under		7 or Over		Winner	
	Tally	Score	Tally	Score	6 or Under	7 or over
Game 1						
Game 2						
Game 3						
Game 4						

1. Which one was the most successful (6 or Under or 7 or Over)?

2. Is this game fair? Explain.

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

[illegible]

[illegible]

3. If 6 or Under is to score a point, there are 6 possible sums that will satisfy the situation. The successful sums for such a situation are 1, 2, 3, 4, 5, or 6. However the probability of obtaining a sum of ONE in tossing 2 dice is zero. Why?

   Any such outcome which has a probability of zero is called an IMPOSSIBLE EVENT.

4. How many different outcomes are there for tossing 2 dice?

5. Out of the 36 possible outcomes of tossing 2 dice, how many of these will yield a sum of 2? ________ a sum of 3? ________ a sum of 4? ________ a sum of 5? ________ a sum of 6? ________.

6. Therefore the probability of obtaining a sum of 2 = 1/36.
   The probability of obtaining a sum of 3 = 2/36.
   The probability of obtaining a sum of 4 = 3/36.
   The probability of obtaining a sum of 5 = _____.
   The probability of obtaining a sum of 6 = 5/36.

   Since ONLY ONE of the sums 2, 3, 4, 5, or 6 can occur at a time to yield a point for 6 or Under, the 5 probabilities are added which yield a total probability of 15/36. Therefore less than 1/2 of the outcomes in tossing 2 dice will yield a point for 6 or Under.

7. If 7 or Over is to score a point one of the following 6 sums must occur: 7, 8, __, 10, __, 12.

   The probability of obtaining a sum of 7 = 1/6.
   The probability of obtaining a sum of 8 = ___.
   The probability of obtaining a sum of 9 = ___.
   The probability of obtaining a sum of 10 = 1/12.
   The probability of obtaining a sum of 11 = ___.
   The probability of obtaining a sum of 12 = 1/36.

   Since ONLY ONE of these sums CAN occur at a time to yield a point for 7 or over, therefore you should (add, subtract, multiply or divide) the following probabilities:

   ________ + 5/36 + 1/9 + ________ + 1/18 + ________ = 21/36.

8. Which one has a better chance of winning: 6 or Under or 7 or Over? Explain.

9. How does this compare with your results as in Table 1? In other words did 7 or Over win instead of 6 or Under? If this was not true, give an explanation.

APPENDIX E

SAMPLES OF NON-DIRECTED LABORATORY LESSONS

SAMPLE LESSON 1

PICK YOUR CHOICE

Materials: 2 different colored dice and a pink and blue gameboard.

Two players are to play this game.

1. There are 15 squares, some are blue and some are pink on a large cardboard.
2. Flip a coin. The winner chooses the color he prefers and will toss the 2 dice while the other person fills in Table 1.
3. Toss 2 dice. Make the same number of moves in the diagram as the sum of the dots on the two dice.
4. If you end on a pink - give pink one point.
   If you end on a blue - give blue one point.
5. On each toss of the dice, always start counting at "start".
6. Toss the dice 20 times (1 game).
7. Repeat this 4 times (or play 4 games).

Table 1

	Tally for Pink	Tally for Blue	Score	
			Pink	Blue
Game 1				
Game 2				
Game 3				
Game 4				

Is this game Fair? Explain.

1. (a) What is the highest number you can throw with two dice? ______

   (b) The lowest? ______

2. How many squares are there on the game board? ______

3. (a) Do you need that many? ______ How many do you need? ______
   (b) Which ones can you land on when you throw the dice?
   ____

4. When throwing two dice do you have a better chance of winning with a sum of 4 or 7? Explain.

   ____

   ____

5. Perhaps the following table will help you answer the above question. Complete Table 2. The first number stands for the outcome on the green die and the second number stands for the outcome on the red die.

Table 2

Die 1 (red)

Die 2 (green)	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

6. (a) Will a sum of 7 land you on the blue? ______
   (b) Will a sum of 12 or 2 land you on pink? ______

7. How many different ways can you throw two dice? ______

8. Out of the 36 possible outcomes of throwing 2 dice, which of these will yield a point for blue? Shade in Table 2.

   ____

   ____

9. Which outcomes will yield a point for pink? ______

   ____

10. Again, is this a fair game? Explain. ______

   ____

   ____

SAMPLE LESSON 2

JIM'S DILEMMA

MATERIALS: chips of 4 colors or chips of 3 colors plus cards.

PROBLEM: Five boys, Jim, Peter, Pat, Harry and Bruce, from Jack Pike High School are the winners at the Provincial Track Meet Finals. They are to be photographed for the local paper. The photographer told the boys to stand in any order in a line they wished. After a minute, the photographer decided that the tallest boy should stand in the middle. Again the photographer wasn't satisfied that this was the arrangement he wanted, and after a minute decided that the shortest boy should be in the middle. Again, the photographer wasn't sure that he wanted to photograph the boys in this order and changed the order of the boys. Remember each change takes one minute.

Jim whispered to Bruce, "Gosh, if the photographer keeps rearranging us, we might be here for two hours before he takes the picture?" Could Jim's statement be correct? Explain.

If you are having problems, do the exercises below and then try the above problem again.

(1) Take 2 different objects, (say 1 white chip and 1 blue chip). In how many ways can you arrange these objects? List them.

(2) Take 3 different objects (say 1 white chip, 1 blue chip and a card). In how many ways can you arrange these objects? List them.

(3) Take 4 different objects. In how many ways can you arrange these objects? List them.

You should have noticed that as the number of different objects increases, the number of arrangements increase rapidly. Therefore, to arrange 5 different objects may take you hours, depending on your speed, and therefore we should see if we can develop a rule for finding the number of arrangements of different objects.

You should have obtained the following:

For 2 different objects - 2 arrangements
For 3 different objects - 6 arrangements
For 4 different objects - 24 arrangements

By using 2, 3, 4 different objects and their respective arrangement of 1, 6, 24, form a rule.

<u>RULE</u>:

4. (a) By arranging 2 different objects:

No. of ways you can choose first object	No. of ways you can choose second object
2 choices	1 choice

$\therefore$ 2 x 1 = 2

(b) By arranging 3 different objects:

No. of ways you can choose first object	No. of ways you can choose second object	No. of ways you can choose third object
3		1

= 3 x 2 x 1 = 6

(c) By arranging 4 different objects:

No. of ways you can choose first object	No. of ways you can choose second object	No. of ways you can choose third object	No. of ways you can choose fourth object
4			1

= ___ x ___ x <u>2</u> x ___ = 24

(d) How many ways can you arrange five different objects?

5. Going back to the original problem, could Jim have been correct in his statement? Explain.

6. 2 x 1 could be expressed as 2! and read as <u>two factorial.</u>

   3 x 2 x 1 could be expressed as 3! and read as <u>three factorial</u>.

   4 x 3 x 2 x 1 could be expressed as 4! and read as <u>four factorial</u>.

   where the symbol " ! " is read as <u>factorial</u>.

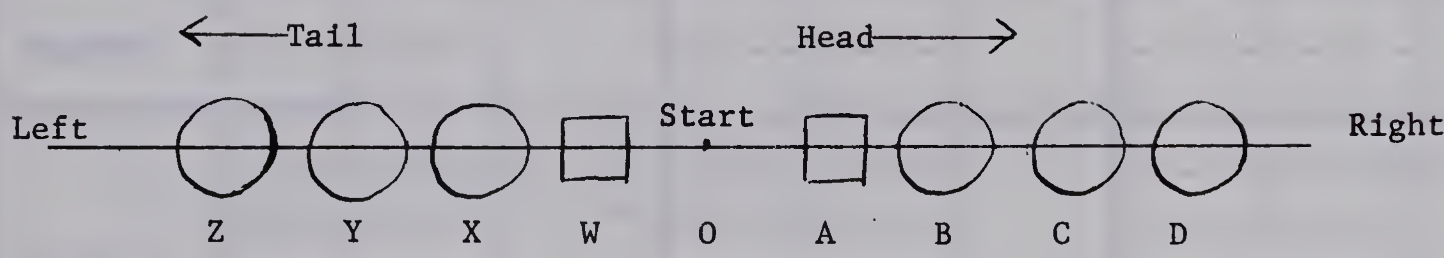
   Therefore 5 ! is read as ________________________________

   and is equal to ____.

SAMPLE LESSON 3

GRASSHOPPER GAME

MATERIALS: 3 different coins: penny, nickel, dime



The circles and squares are represented by letters at the bottom of these positions.

Two People Are To Play This Game.

Rules:

1. Flip a coin, the winner chooses the position he prefers (either circle or square).

2. Each play begins at "O" (START) and then toss the three coins,(penny, nickel dime).

3. Notice the way the coins land, for each head move one to the right, for each tail move one to the left (i.e. if coins land THH, your moves would be O to W to O to A and therefore one point for squares. If on next toss coins landed HHH, your moves would be O to A to B to C and a point for the circles.

4. One person is to flip the coins, the other is to record the results in Table 1.

5. 16 plays (16 tosses of 3 coins) make up a game.

6. Play 3 games.

7. After each game, total your scores. The one with the most tallies wins.

8. NOTE: Frequency means the number of times something occurs.

Table 1

		GAME 1			GAME 2			GAME 3		
		Tally	Fre-quency	Score	Tally	Fre-quency	Score	Tally	Fre-quency	Score
Squares	A									
	W									
Circles	Z									
	Y									
	X									
	B									
	C									
	D									

Table 2 (For all 3 Games)

Place Play Ends	Frequency (Total for 3 games)	Fraction of Total Plays
A		
W		
Z		
Y		
X		
B		
C		
D		

Total =

The fractions should add up to 1.

1. Do you think this game is fair? Why?

2. If you played this game again, what shape would you choose? Why?

[illegible]

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

[illegible]

[illegible]	[illegible]	[illegible]

[illegible]

[illegible]

[illegible]

3. Why is the frequency for each of Z, X, D, B = 0? Explain.

4. What are the different ways in which the three coins may land? List them.

5. What are 6 ways out of a total of 8 possible ways the coins land that will lead to squares getting a point?

6. Probability is defined as $\frac{\text{no. of favorable outcomes}}{\text{total no. of possible outcomes}}$.

   Calculate the probabilities for obtaining each of Z, Y, X, W, A, B, C, D, and compare these with the ratios in Table 2. Have you any explanation for the differences? Since you are calculating the probabilities for all possible outcomes, the probabilities should add to 1.

7. For 128 trials in the Grasshopper game, and using your results from probabilities in question 6, how many trials would end at:

   (a) W ______________ (e) A ______________

   (b) X ______________ (f) B ______________

   (c) Y ______________ (g) C ______________

   (d) Z ______________ (h) D ______________

   Did you get 48, 0, 16, 0, 48, 0, 16, 0? Do these add up to 128? What conclusions can you make?

8. How does the total of your scores (from Table 2) in the ratio number of squares to number of circles compare to the calculated probabilities of $\frac{A + W}{Y + C}$? If these ratios differ, give an explanation.

9. How would you make this game fair?

SAMPLE LESSON 4

$P \cap Q$ - Die Vs Coin

MATERIALS: a die and a coin

Two players are to play this game.

Rules:

1. There are two possible ways of scoring a point:

   2 routes
   - (a) Tossing a die twice and getting a four or less on BOTH tosses.
   - (b) Flipping a coin twice and getting a head on BOTH flips.

2. Flip a coin, the winner plays first. He chooses one of the two routes as outlined in number 1 above. If he is successful on one try, he scores a point. He is to repeat this 10 times.

3. The other player now chooses the other route and repeats his 10 times. When both players have had 10 chances, this will make up one game.

4. Play 4 games.

5. Fill in Table 1.

TABLE 1

	Person 1			Person 2			Winner	
	Route			Route				
	Die	Coin	Score	Die	Coin	Score	Die	Coin
Game I								
Game 2								
Game 3								
Game 4								

1. Did you find one route leading to more successes than the other? Can you account for this?

2. How many possible outcomes are there when a coin is flipped once?

3. Out of the two possible outcomes, the probability of getting a tail is 1/2. What is the probability of obtaining heads?

4. In flipping the coin a second time, what is the probability of obtaining a head?

5. The probability of obtaining a head on first slip = 1/2.
The probability of obtaining a head on second flip = 1/2.
Since 2 heads MUST appear consecutively, the probability of getting 2 heads in a row is $\frac{1}{2} \times \frac{1}{2} = 1/4$. Notice you MULTIPLY the probabilities of the separate events because BOTH events must occur.

6. In tossing a die once how many possible outcomes are there?

7. Out of the 6 possible outcomes, how many are favorable (according to the game just played)?

8. What is the probability of obtaining a success on the first toss?

9. Out of the 6 possible outcomes on the second toss of the die, what fraction of these will lead to a point (as described in the game)?

10. The probability for obtaining 4 or less on first toss = 4/6 = 2/3.
The probability for obtaining 4 or less on second toss = 2/3.
Are you going to (multiply, add, subtract, or divide) the 2 probabilities above? Explain.

11. You have noticed that the probability of obtaining a 4 or less on both tosses of a die is 2/3 x 2/3 = ___________.

12. The probability of getting 2 heads in a row is 1/4. The probability of getting 4 or less in 2 tosses of a die ≐ 4/9. Which one has the better chance of winning? Why?

13. Did the die in Table 1 have more points than the coin? If not, try to explain why this happened?

14. Which of the following would you choose?

    (a) Toss a die twice, must get a 4 or less on both tosses.
    (b) Toss a die three times, must get a 5 or less on all three tosses.

    Take a die and toss it twice - repeat 20 times. How many times did you have a success? ___________ Then toss the die 3 times - repeat 20 times - how many successes? ____________ Calculate the probability of (a) and (b). Did you get 4/9 and 125/216? Which probability is larger?

SAMPLE LESSON 5

6 OR UNDER, 7 OR OVER

MATERIALS: 2 different colored dice (red & green)

Rules:

Two players are to play the following game.

(1) Toss two dice 20 times. This will make up a game.

(2) Notice the sum of the 2 dice on EACH toss.

(3) There are 2 possible ways of scoring a point.

(a) 6 or Under - if the dots on the two dice add up to 6 or less, score a point for the person who chose this situation.

(b) 7 or Over - if the dots on the two dice add up to 7 or over, score a point for the person who chose this situation.

(4) Flip a coin. The winner chooses the situation for scoring a point he prefers as outlined in number 3 (above). The other person gets the other way of scoring points.

(5) Play 4 games.

(6) Fill in Table 1 as you play each game.

TABLE 1

	6 or Under		7 or Over		Winner	
	Tally	Score	Tally	Score	6 or Under	7 or Over
Game 1						
Game 2						
Game 3						
Game 4						

1. Which one was the most successful (6 or Under or 7 or Over)?

2. Is this game fair? Explain.

3. If 6 or Under is to score a point, there are 6 possible sums that will satisfy the situation. The successful sums for such a situation are 1, 2, 3, 4, 5, or 6. However the probability of obtaining a sum of ONE in tossing 2 dice is zero. Why?

   Any such outcome which has a probability of zero is called an <u>IMPOSSIBLE EVENT</u>.

4. How many different outcomes are there for tossing 2 dice?

5. Out of the 36 possible outcomes of tossing 2 dice, how many of these will yield a sum of 2?_________, a sum of 3?__________ a sum of 4?______, a sum of 5?___________, a sum of 6?___________.

6. Therefore the probability of obtaining a sum of 2 = 1/36.
   The probability of obtaining a sum of 3 = 2/36.
   The probability of obtaining a sum of 4 = 3/36.
   The probability of obtaining a sum of 5 = _____
   The probability of obtaining a sum of 6 = 5/36.

   Since ONLY ONE of the sums 2, 3, 4, 5, or 6 can occur at a time to yield a point for 6 or Under, the 5 probabilities are added which yield a total probability of 15/36. Therefore less than 1/2 of the outcomes in tossing 2 dice will yield a point for 6 or Under.

7. If 7 or Over is to score a point one of the following 6 sums must occur: 7, 8, __, 10, __, 12.

   The probability of obtaining a sum of 7 = 1/6.
   The probability of obtaining a sum of 8 = _____.
   The probability of obtaining a sum of 9 = _____.
   The probability of obtaining a sum of 10 = 1/12.
   The probability of obtaining a sum of 11 = _____.
   The probability of obtaining a sum of 12 = 1/36.

   Since ONLY ONE of these sums CAN occur at a time to yield a point for 7 or Over, therefore you should (add, subtract, multiply or divide) the following probabilities:
   _____ + 5/36 + 1/9 + _____ + 1/18 + _____ = 2/36.

8. Which one has a better chance of winning: 6 or Under or 7 or Over? Explain.

9. How does this compare with your results as in Table 1? In other words did 7 or Over win instead of 6 or Under? If this was not true, give an explanation.

A P P E N D I X   F

TEACHER'S GUIDE

APPENDIX F

TEACHER'S GUIDE

RATIONALE

In the last decade educational research in the field of mathematics has mainly focused attention on the average and above average, college-bound student. The low achiever is only now beginning to be recognized. For this reason, we have designed a unit on probability in conjunction with the Math 15 program. This unit emphasizes the following principles.

1. CONCRETE MATERIALS - Several well-known psychologists such as Piaget and Bruner have suggested that learning new concepts in its initial stages can best be done with the aid of concrete materials; therefore, all the lessons in the unit will allow the student to manipulate concrete materials.

2. SOCIAL PARTICIPATION - The students will work in groups of two or three since interaction between individuals can help speed up the process of concept attainment.

3. MULTIPLE EMBODIMENT - This principle implies that each concept be taught in several ways. For this reason there are several laboratory lessons for each of the concepts.

4. SHORT LESSONS - The lessons are deliberately kept short to help maintain the interest of the students. An attempt is made to use everyday language. However, you should expect to help students interpret in a few places and students should expect to read what there is to read.

5. INDIVIDUALIZATION - Since students work at different rates, optional lessons have been written for the faster students. Therefore, NOT ALL of the students will do ALL of the lessons.

6. DISCOVERY AND NATURE OF THE LESSONS - The lessons are designed to allow the student to discover the mathematics concept for himself as much as possible.

Two types of laboratory lessons have been developed - the directed and non-directed lab. The essential difference between these two labs is that in the directed lab the students are given some mathematical background about the particular lesson before playing the game or trying the problem of that particular laboratory lesson. In the non-directed laboratory lessons the students play the game or try the problem first and then they try to extract the mathematics behind the lab.

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

PROCEDURE

1. Do NOT be confused by the names directed and non-directed laboratory. The way in which you act in both lab settings should be the SAME.

2. Please explain to the students that both labs contain equal amounts of work but are just written in a different manner.

3. Group the students in pairs according to friendship criteria. NOTE: It is important that one member of the group can read.

4. There are two types of lab booklets. Randomly assign half of the group to one type of lab and the other half to the other type. (Please make sure that all the "best" students don't get one type of lab).

5. Assign each group one booklet. Each booklet contains the written instructions for about 2 - 3 weeks of work.

6. The students are required to complete all the lessons except those which are optional. The optional lessons are provided for those students who work faster than the rest of the group. They provide further reinforcement of the concepts that are to be learned.

7. There are 5 basic concepts in the booklet. Each concept is printed on a different color of paper so that the student and teacher can easily find the section they are on.

8. Because the booklet is short term, this method need not be a true individualized approach. The teacher might wish to conduct a class discussion on topics that the students are having problems with.

9. You might expect more noise in the classroom because the students will be working in pairs. However, this has not proven to be a problem previously if the students are working.

10. NOTE: These activity-oriented labs require that the teacher be active. The lessons are NOT programmed learning so that students will need to be taught parts of the lesson. The teacher is to act as a guide to the pupils and NOT to leave them on their own during the duration of the lab lessons.

11. The class should take up the questions at the end of each section. (The questions are on the white pages.) Here the teacher might use a more conventional method of teaching.

12. Students can check their answers for the lab lessons from the teacher's manual if they wish.

HOW TO INTRODUCE THE MATERIAL

Teacher

1. For the initial introduction demonstration ask, "How many different ways do you (the class) think a deck of cards can be shuffled?" After a discussion where the students guesses are recorded they might be interested in the following information. "The total of different sequences possible in a 52-card deck is a figure 68 numerals long; if all the people on the earth counted a million arrangements a second 24 hours a day for 80 years, they could not count a billionth of a billionth of 1 per cent of the possibilities. The total number of 5 card poker hands possible is 2,598,960. The number of 13-card bridge hands: 635,013,559,600". (page 139, Life Science Library - Mathematics).

2. Read with the students the introduction to probability.

   (a) The question on page 2 is a brainstorming question to build up some enthusiasm. See who can put the most occupations down in 3 minutes and then discuss.

   (b) Then continue reading and do the newspaper questions. Try to get the students to talk freely.

3. Explain to the students about how they are going to be working in pairs for the next two to three weeks and that each pair is going to have one booklet which cannot be taken from the classroom.

APPENDIX G

CONCRETE MATERIALS

APPENDIX G

CONCRETE MATERIALS

Each show box, used by a group of students, contained the following concrete materials.

1. 100 poker chips colored red, blue and white
2. a cube whose sides are colored white, green, blue and red
3. 30 orange colored cards
4. 30 red colored cards
5. 30 blue colored cards
6. 30 yellow colored cards
7. 30 green colored cards
8. 2 paper bags
9. 2 die -- 1 green and 1 red
10. a gameboard card consisting of 9 red colored squares and 6 blue colored squares.